University of Twente
Enschede - The Netherlands

Route choice modelling in dynamic traffic assignment

OmniTRANS
Transport Planning Software

MSc thesis, Peter Kant
Deventer/Enschede
July 2008

Peter Kant

Route choice modelling in dynamic traffic assignment
Master thesis

Date July 4, 2008

Documentation page

Title report	Route choice modelling in dynamic traffic assignment

Keywords Dynamic Traffic Assignment, Route choice modelling

Author Peter Kant
p.kant@alumnus.utwente.nl

Committee members Prof. Dr. Ir. E.C. van Berkum
University of Twente
Centre for Transport Studies
e.c.vanberkum@ctw.utwente.nl

Dr. W. Kern
University of Twente
Discrete Mathematics and Mathematical Programming
w.kern@math.utwente.nl

Drs. H.E. Mein
Omnitrans International
emein@omnitrans.nl

Ir. D.H. van Amelsfort
Goudappel Coffeng
dvanamelsfort@goudappel.nl

Date of publication July 4, 2008

Summary

Problem definition

Existing dynamic traffic assignment models become more and more advanced in terms of propagation, but in terms of route choice, many models are still pretty primitive. With the increase of research for route choice models, the question arises if it is possible to extend current propagation models by adding route choice to them. Questions included are which route choice models are good enough for application and how these models should be combined with the propagation model.

Methodology

In recent years, the research on route choice models and –more in general– discrete choice models from Random Utility Maximisation has increased significantly. Many research has been performed from a theoretical point of view. In this research the performance of several GEV based models is tested using a Monte Carlo (Probit) simulation technique. For this purpose a large scale network is used out of which 26 zones are selected for routeset generation, filtering and route choice calculation.

Additional research is performed to determine how route choice models and propagation models have to be combined. A new flexible dynamic equilibrium is presented to replace two existing dynamic equilibria (Boston & DUE). This equilibrium is formulated to better approach real traveller behaviour. A prototype model is developed and tested on a relatively small network.

Results and discussion

There are significant relations between characteristics of routesets and performance of the tested route choice models. This implies that the question which route choice models should be applied depends largely on the type of routeset. In general the PCL model gives relatively good results while not much effort has to be put in calibration.

The prototype model for interaction between route choice and propagation shows results matching expectations. Combining iterations using a method of successive averages leads to fast and stable convergence. If models get very stochastic, the ability to optimise route choice decreases.

Conclusion and recommendations

It is possible to extend existing propagation models with route choice. The PCL route choice model is a good place to start, although also CNL, PSL and C-Logit give good results, depending on the characteristics of the routeset. If possible it is advised to determine which model to use before application.

The new equilibrium method using some kind of forecasting is assumed to be very powerful. It gives more flexibility than current instantaneous and dynamic equilibria.

Tests performed with the prototype are promising. However, additional research is needed to determine if the results presented in this report can be generalised to larger cases. Especially the applicability on large scale networks (with congestion) is advised to be investigated. For this purpose, the current prototype can be optimised to a more efficient test application.

Samenvatting

Aanleiding en probleemstelling

Bestaande dynamische toedelingsmodellen worden steeds geavanceerder op gebied van propagatie, maar zijn vaak nog erg beperkt in de routekeuze. De vraag die aan de basis van dit onderzoek ligt is hoe routekeuze op een juiste manier aan bestaande dynamische toedelingsmodellen kan worden toegevoegd.

Methodiek

Routekeuze wordt in de literatuur voorgesteld vanuit de Random Utility Maximisation Theory. De afgelopen jaren zijn verschillende modellen voor routekeuze ontwikkeld, maar vaak enkel van een theoretisch perspectief. In dit onderzoek zijn de modellen aan een test onderworpen door ze toe te passen op een grootschalig netwerk van Nederland. Voor 26 zones in dit netwerk zijn routesets gegenereerd. Routefracties zijn berekend via een Monte Carlo (Probit) simulatie waarop de routemodellen zijn gekalibreerd. Vervolgens zijn de modelresultaten voor routesets met specifieke eigenschappen vergeleken.

Aanvullend is onderzocht hoe de routekeuzemodellen aan de propagatie-modellen gekoppeld dienen te worden. Een nieuwe evenwichtsdefinitie is geformuleerd die een combinatie van een instantaan evenwicht en dynamisch evenwicht mogelijk maakt en daarmee beter aan kan sluiten op reizigersgedrag uit de praktijk. Hiervoor wordt een soort van voorspellingsalgoritme gebruikt. Deze methodiek is omgezet in een prototype en getest op een kleinschalig netwerk.

Resultaten en discussie

Er is een relatie tussen eigenschappen van een routeset en de nauwkeurigheid van de routefracties zoals uitgerekend door de keuzemodellen. Dit maakt dat verschillende keuzemodellen beter aansluiten op bepaalde situaties. In het algemeen geldt dat het PCL model het makkelijkste te kalibreren is en daarmee het meest eenvoudig toegepast kan worden. Daarbij geeft het voor een groot deel van de routesets goede resultaten.

Het prototype van het model dat de interactie tussen routekeuze en propageren regelt, laat zien dat alle parameters van het model de verwachte resultaten opleveren. Een systematiek van opeenvolgende gemiddelden voor meerdere iteraties leidt tot snelle en effectieve convergentie. Hoe deterministischer een model is, hoe makkelijker convergentie wordt bereikt.

Conclusie en aanbevelingen

Geconcludeerd wordt dat het goed mogelijk is om bestaande propagatiemodellen te voorzien van routekeuze. Het PCL routekeuzemodel is een goed alternatief voor eenvoudige implementatie, maar ook CNL, PSL en C-Logit zijn goede alternatieven, afhankelijk van de karakteristieken van de routeset. Indien mogelijk wordt geadviseerd om het gebruik daarom af te stemmen op de routeset.

De methodiek waarbij routekeuze plaatsvindt op basis van een soort voorspelling wordt als zeer waardevol beschouwd. Het biedt meer flexibiliteit dan de bestaande instantane en dynamische evenwichtsformulering.

De tests die met het prototype model zijn uitgevoerd zijn veelbelovend. Desalniettemin is aanvullend onderzoek nodig om te bepalen in welke mate de huidige resultaten gegeneraliseerd kunnen worden. Vooral de toepasbaarheid op grootschalige netwerken, al dan niet met congestie, is een punt van onderzoek. Daartoe wordt aangeraden het prototype efficiënter te formuleren en toe te passen op realistische netwerken met een zekere graad van congestie.

Preface

This thesis is the final work of my graduation study at the Centre for Transport Studies, department of Civil Engineering and Management, University of Twente. The corresponding research has been conducted at Omnitrans International.

Graduating is like making a big journey. Leaving high school, through the bachelor stage with the final goal to obtain a master degree. Such a trip requires a path in which route choice is necessary but far from easy. Thereby it is comparable to modelling transport: with route choice the results can be far better than without.

As a child I always wanted to build bridges, which actually was the reason I chose to study civil engineering. Along the way, bridge building has changed from engineering to a metaphor for linking two different worlds. The multidisciplinary character of civil engineering fits this objective very well.

When Erik de Romph gave me the opportunity to undertake my research at Omnitrans, the decision was easy to make. To me it felt like the perfect environment to link transportation science and programming, another hobby I have had for a long time. During a 9 month period I have been able to not only conduct my research on route choice, but also to be part of the MaDAM redevelopment team and to see how interesting working at a transport modelling company is.

I would like to thank my professors (Eric & Walter) and coaches (Edwin & Dirk) for their guidance and help to finish my work successfully. Further I want to thank my colleagues at Omnitrans. They made each day enjoyable and above all inspired me to work on transport modelling. John, I am grateful for your remarks along the way that helped me to improve this thesis. Then I would like to thank my family, especially my parents, for always supporting me and for giving me the opportunity to realise my objectives. Last, but definitely not least I want to thank my friends, without them those 6 years of study would have been less interesting. Special gratitude goes to Maarten, Erik, Everhard and Ties for supporting me with this research.

Deventer/Enschede, June 2008

Peter Kant

Contents

1 Introduction

For only a few people is transport an end in itself. Most need to travel to be able to perform a certain activity, e.g. work, education or shopping. Since activities are carried out at different locations, people tend to make trips between these locations. This results in mobility and has its effect on daily society life. This chapter gives an introduction to the study presented in this report on dynamic traffic assignment route choice modelling. Paragraph 1.1 gives a short introduction on the growing mobility that partly forms the research motive pointed out in paragraph 1.2. In paragraph 1.3 the research objective and questions are presented. Paragraphs 1.4 and 1.5 describe the context of the study and scope of research respectively. This chapter concludes in paragraph 1.6 by giving an outline of the report contents.

1.1 The need for transportation modelling

For the Netherlands, the last decade has shown a significant increase in mobility. Prospects for 2020 show a further increase of approximately 20% compared to 2000. Especially the mileage for car-drivers will grow (Ministry of Transport and Public Works, 2004). Without investment in new roads, this will lead to more congestion. This growth of mobility places pressure on the quality of life and environment. The trend of increasing mobility is not limited to the Netherlands. In both developed and developing countries this phenomenon requires attention.

Comprehensive policies are required to cope with the growth of mobility. Transport planning and traffic management are necessary to support policy development and evaluation. Traffic management particularly focuses on the interaction between travel supply and demand. Behavioural information of the traffic is monitored by the traffic manager who regulates and controls the traffic. This is like a ‘two-way interaction game’ (Bovy & Stern, 1990).

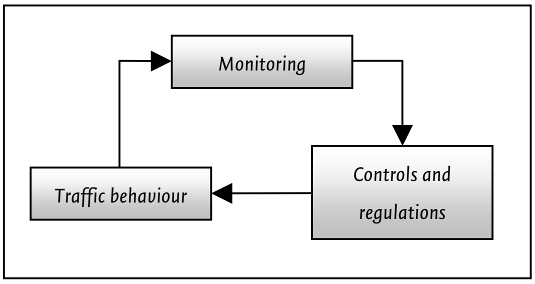


Figure 1.1 A basic traffic management approach (after Bovy & Stern, 1990)

To improve policy making and policy evaluation transport models are used. These models can be used to determine the effects of traffic measures. Offline models are used to forecast traffic demand and flow patterns based on zonal data and empirical data like traffic counts. Online models use real-time data. For this research offline models are considered.

A common modelling approach consists of the four steps of trip generation, trip distribution, modal split and traffic assignment. For the traffic manager, especially the assignment stage is interesting, because it supplies in estimation of the effects of (dynamic) traffic measures before they are applied in real-life. For a comprehensive overview of the traffic assignment problem and related issues, see Patriksson (1994).

1.2 Research motive

Dynamic Traffic Assignment (DTA) models can be used to estimate the network load over time based on dynamic travel demand. Since the research of Merchant and Nemhauser in the late 1970s - which may be considered as the basis for all dynamic traffic assignment models - DTA modelling has evolved many times (Bliemer, 2001). However, it is still considered as relatively undeveloped (Peeta & Ziliaskopoulos, 2001).

DTA models contain two interdependent components: route choice and dynamic network loading. Route choice models determine the behaviour of flows in the network. Dynamic network loading (DNL) describes the flow propagation through the network. A distinction can be made for two types of DTA models.

The first type uses one comprehensive framework in which route enumeration and flow propagation is performed merely analytical. Relatively easy link performance functions are used, e.g. linear link exit functions (Bliemer, 2001). This simplicity enables the model to use advanced existing mathematical techniques to solve the DTA problem. The realism of traffic propagation thereby is of secondary importance (Szeto, 2003). The benefit of this approach is that existence and uniqueness of a solution can be proven (Yperman, 2007).

The second type of models are simulation based models. These models use iterative procedures to derive the dynamic flow pattern. The mathematical techniques used by this type of models are less analytically oriented compared to the 'analytical' type models. Link performance functions are less restricted and can take complex forms. The DNL (sub)model supports advanced, non-linear fundamental diagrams and queue spillback models.

A problem with many of the existing DTA models is that they often focus on either route choice or dynamic network loading. Especially the models with advanced DNL (sub)models lack the implementation of route choice. However, DNL models recently received more attention, merely because of their improved ability of capturing flow dynamics (Bliemer, 2001).

The presence of good DNL models raises the question whether such models could be extended to full DTA models. This requires solving the route choice problem and implementing the interaction between route choice and dynamic network loading. The literature does give theoretical information on the first subject, but practical information is not widely available. Further the interaction between route choice and dynamic network loading is a subject that deserves more attention.

1.3 Objective and research questions

The objective of this study is to contribute to transport modelling by presenting a framework that extends existing DNL models with route choice.

> *"The aim of the study is to develop a route choice model as an extension for current macroscopic DNL models, taking into account the interdependence of route choice and network loading."*

In order to support the accomplishment of the objective two research questions are formulated.

1. What are the specifications of a route choice model as part of a dynamic traffic assignment model?
2. How can the iterative characteristic between dynamic network loading and route choice be realised?

1.4 Research context

The study is undertaken at Omnitrans International in Deventer, The Netherlands. The DTA model MaDAM, based on the METANET DNL model was developed by Omnitrans in the 1990's. This model is currently under redevelopment. One of the aims in the redevelopment process is to improve the process of route choice modelling. This study therefore refers to MaDAM occasionally. More information about the MaDAM model can be found in paragraph 2.4.

1.5 Scope of study

Before exploring the contents of this report, it is important to note what is part of the research presented and what is not.

Type of assignment
The study is focussed on macroscopic dynamic traffic assignment. Occasionally there are some references to static assignment techniques.

Level of detail
The study is used for the development of a route choice model to be used in medium to large scale networks, comprising up to 4000 zones and 200.000 links. When defining the route choice model and the interaction between route choice and network loading, this order of magnitude is considered, which means the level of detail is relatively limited.

Departure time modelling
Departure time modelling is not under investigation in this research.

Modes and purposes
Only one travel network is considered: the motorway network. The number of modes and purposes is limited to comply with the desired level of detail: The model abstracts to only 1 mode and different types of network users (and so purposes) are represented by user classes.

Propagation modelling
Although propagation modelling is an important component in DTA modelling, it is not under investigation in this report.

Data use and calibration
The aim of the study presented is to develop a general model for route choice in a dynamic traffic assignment context. Therefore, calibration and validation is not described in this report.

1.6 Report outline

This research is structured as follows. Chapter 2 gives an introduction to traffic assignment; first explaining static assignment and then presenting the reasons for dynamic traffic assignment.

In chapter 3 a conceptual framework is presented that contains all elements that are elaborated in the chapters 4-7. First chapter 4 discusses route choice and routesets from a theoretical perspective.

In chapter 5 route choice is presented as a discrete choice problem. Random utility maximisation theory is used to express route choice models from both the Logit and Probit family. A central focus is on special route choice models developed in recent years. These models are analysed in chapter 6. A selection from a real network is used to simulate model performance and see how well the models perform.

Chapter 7 focuses on the interaction between route choice and dynamic network loading. Solution algorithms are investigated and presented mathematically. Chapter 8 will test a new algorithm using a case study.

Finally, in chapter 9 the findings from the previous chapters are summarised in the conclusions. Also an outline is given of possibilities for further research.

2 Traffic Assignment overview

This chapter focuses on dynamic traffic assignment from both a theoretical and practical perspective.

➥ *In paragraph 2.1 traffic assignment will be introduced in general, followed by discovering different types of traffic assignment in paragraph 2.2. Paragraphs 2.3 and 2.4 focus on static and dynamic assignment respectively. The chapter ends with paragraph 2.5 on the MetaNET/MaDAM model.*

2.1 Introduction

Traffic assignment is the fourth stage in the classic four-step transport planning approach. As described by Ortúzar and Willumsen (2001), the assignment model determines an optimal trade-off between supply and demand, based on some given decision rules. The decision rules include the route choice behaviour of the travellers in the network. In practice, it means all trips are assigned to the network resulting in a traffic flow pattern.

Compared with the network's infrastructure, the resulting flow pattern gives information about the performance of the network. For the traffic manager this is necessary information when determining effective management techniques.

2.2 Types of traffic assignment

Assignment models are subject to the assumed traveller behaviour and network performance. If the model takes into account delays due to travel demand, the model is capacity restrained. Travellers route choice depends on the costs of the available routes. If the travellers are assumed to have perfect knowledge of the network conditions, a full equilibrium is simulated. If perception differences are simulated, the assignment model yields a stochastic flow pattern.

		Stochastic effects included?	
		No	Yes
Capacity restraint included?	No	All-or-Nothing	Pure stochastic
	Yes	Wardrop's Equilibrium	Stochastic User Equilibrium

Figure 2.1 ***Types of traffic assignment***
(after Ortúzar and Willumen, 2001)

2.3 Static traffic assignment

2.3.1 Static equilibrium definitions

Based on the classification scheme in figure 2.1 the following static equilibria are distinguished.

Wardrop's First Principle (User optimum)

The first principle states that under equilibrium conditions, no individual trip maker can reduce his path cost by switching routes. This means that all used routes between an origin and destination have equal impedances and all declined routes have larger impedances. This principle is also known as the User Optimum.

Wardrop's Second Principle (System optimum)

Wardrop's second principle, also known as the System Optimum, defines a state in which the network total travel cost is minimised. This means no single traveller can change routes to reduce his costs without thereby increasing the travel costs of other travellers (Wardrop, 1952).

Stochastic User Equilibrium

Under equilibrium conditions, no individual trip maker *believes* he can reduce his path cost by switching routes.

2.3.2 Static assignment algorithms

Although other algorithms are possible (and gaining attention), widely used static assignment algorithms are based on repeated shortest path searches and dynamic network loading. The techniques differ in the way they assign the traffic to the network: incrementally, by convex combination or by using a line-searching technique (Frank-Wolfe algorithm). If a solution flow pattern exists, such methods will converge to this solution[1].

For a comprehensive exploration of (mathematical) assignment algorithms, see Sheffi (1985) and Patriksson (1994).

2.3.3 Uniqueness of solution

If an equilibrium flow pattern exists and is unique, this solution is by definition only unique in terms of link flows. Multiple route flow patterns may result in the same link flow pattern and therefore there is not by definition a unique route flow pattern that results in a equilibrium traffic situation.

[1] Whether the equilibrium flow pattern is derived depends on the used stepsize.

2.3.4 Drawbacks of static assignment

In the static case travel demand for a certain period is known (for instance a morning peak period). One single OD-matrix contains the trips, which are all assumed to start and end within this period.

Some remarks must be made on static assignment. First, recalling from the previous paragraph an equilibrium solution – if one exists – exists only in terms of link flows and not on route level per se (although it is theoretically possible in some small networks). Secondly, for longer trips (most of) the static assignment models do not take into account that trips might not reach their destination within the modelled time period. This aspect has recently received attention in a paper by Clark *et al.* (2007). The largest drawback of static assignment models is that they do not take into account the traffic flow through the network over time.

2.4 Dynamic traffic assignment

DTA models overcome the limitations of static assignment models by using a dynamic network loading model. Such a model uses continuous or discretised[2] time to model traffic flow through the network. Compared to static assignment models, congestion effects are simulated far more realistic.

> *Dynamic Network Loading model (propagation model)*
> *In dynamic assignment modelling, a propagation defines the interaction between traffic on the network, like headway interaction, speed-density relations and stop- and go actions. Several propagation models have been developed over time, varying from car-following theory to kinematic wave and gas flow theory.*

2.4.1 Application of assignment models

In the past static assignment techniques have been used on a large scale. During the last decade a shift toward more use of dynamic models can be seen. There are two major causes for this trend. First, the demand from the market has changed. For a long period there was practically no need for dynamic assignment models, since static models gave (and still give) robust results for the purpose of transport planning. Over time the market (consultants and their clients) also wanted insight in travel times (and delays) and queue building. This required a model that took time dynamics into account. Secondly, the computational power needed for DTA models is huge and until the late 1990s required special computers. With the increasing possibilities of regular computer workstations the dynamic traffic assignment is gaining importance rapidly (Peeta & Ziliaskopoulos, 2001).

[2] Discretised models use small time steps (e.g. one second).

2.5 The MetaNET/MaDAM model

As described in paragraph 1.4 the study presented here is carried out at Omnitrans International (OTI). The DTA model MaDAM, based on the DNL model METANET, is developed at OTI. Because the findings of the presented study might be used in the process of redeveloping MaDAM, it is important to briefly describe the current MaDAM model.

2.5.1 METANET

The basis for MaDAM lies in the METANET model, developed by Messmer and Papageorgiou in the 1990's. In METANET time and space are discretised. The network is represented by a directed graph, where links are singly directed and the geometrics for a link are assumed homogeneous. Nodes between the links are used as diverge or converge points or at locations in the network where the motorway characteristics change (e.g. number of lanes). A large restriction however is that only nodes of degrees 2 and 3 are possible, which makes it inefficient to model realistic networks (Van Berkum, 2007).

For each link a fundamental diagram is assumed, based on the link parameters free flow speed, speed at capacity, jam density and saturated flow. To propagate traffic, the METANET model divides the links into segments of equal length. All flow variables are calculated for each segment, using the fundamental diagram and using traffic conservation equations to be realistic with the conditions on the segments upstream and downstream.

Route choice behaviour is presented by defining splitting rates for cross- and diverge nodes (nodes with multiple exit links). Both the travel demand and the splitting rates (turn fractions) may change over time, as specified by the user.

2.5.2 MaDAM

There are three major differences between MaDAM and METANET, which makes them two different models. The first major improvement is the ability to use nodes of higher degrees in MaDAM. This makes it possible to model full intersections (i.e. with 4 entering and 4 exiting links).

The second major change is the use of a different fundamental diagram compared to METANET. According to the developers of MaDAM the original fundamental diagram from Metanet gives unrealistic results when volume approaches capacity the speed drop is too large, while they believe in such situations still relatively high speeds can be reached. Therefore, the Van Aerde fundamental diagram is used, which addresses supplies in this concern.

The third and last major difference between MaDAM and METANET is that MaDAM contains a special junction modelling module, which has the ability to calculate delays on intersections.

3 Conceptual framework

The previous chapter has introduced the concept of dynamic traffic assignment. This chapter will present a conceptual framework and gives a short introduction to the chapters 4, 5, 6 and 7.

The DTA framework has to be consistent with existing Dynamic Network Loading models, but add (or replace existing) route choice models. The framework contains a route choice module that is fully flexible with – and operates independently of the used DNL model for propagation.

↪ *Paragraph 3.1 starts with defining the variables used in the framework, followed by a short outline of the framework in paragraph 3.2. The model constraints are investigated in paragraph 3.3. The concepts of route choice and DNL-Route choice interaction are discussed in paragraphs 3.4 and 3.5.*

3.1 Variables

The mathematical formulations of the framework use index characters to define to which elements data is related. The following indices are used.

3.1.1 Location variables

Origin (o)
Zone in the network from where traffic departs.

Destination (d)
Zone in the network where a trip ends.

Route (r)
Series of links from an origin to a destination zone.

Position (x)
A position along a route is denoted by x. This location is route, departure interval and time dependent.

3.1.2 Time variables

General time (T, t)
The framework uses multiple time dimensions. The total (continuous) modelling period is denoted by T. Index t denotes a moment within this period. This index is continuous or discrete, depending on the used dynamic network loading model.

Route choice interval (k)

The total time is divided into several time windows. In each time window the route choice behaviour is equal i.e. all travellers are assumed homogeneous. The length of such a time interval is quite arbitrary. If many small time windows are used, the traveller behaviour tends toward a microscopic model (since only small fractions of vehicles are considered equally behaviouring). k is used as additional index for t.

Forecasting horizon (λ)

A forecasting horizon might be used (more info in paragraph 7.5), where λ is used as additional index for t.

Time aggregation (γ)

Data is aggregated in equal time intervals, where the size of an interval is defined by the modeller.

> *Time-location example*
>
> $x_{r,\delta}^{rod}$ defines the point on route r from origin o to destination d, departing in route choice interval k, δ time units after departure. If $\delta = \infty$ this means the end of the route.

3.1.3 Network variables

Link (a)

Element of the network at which time-dependent traffic conditions are stored.

3.1.4 Traveller variables

Individual (n)

For theoretical modelling an individual traveller n is considered.

User class (u)

A group of travellers who are assumed homogeneous. User classes can be distinct on characteristics like vehicle type and traveller behaviour.

3.2 Framework outline

The conceptual framework is depicted in figure A.1 on the page 67 (this figure can be folded out for viewing while reading). Based on previous steps in the transport model, travel demand is assumed to exist for each time period and user class. Travel cost functions are used to determine the routeset (generate and filter). The main part of the DTA model framework consists of loops in which route choice and dynamic network loading are iteratively optimised to derive the flow pattern meeting a set of constraints.

Multiple user classes

Dynamic traffic assignment models using a discrete choice model for stochastic route choice allow some variation among travellers, but the traveller population is still considered homogeneous. Bliemer (2001) and Rosa & Maher (1999) suggest to extend DTA models with the inclusion of multiple user classes to represent heterogeneous traveller characteristics and thereby increase model applicability and make them more realistic.

In the presented framework multiple user classes are supported for all model elements except the dynamic network loading model. Depending on the DNL model used, user information might not be used during traffic propagation. The framework however is flexible and is still able to evaluate route choice for each user class after propagating.

3.3 Constraints

The dynamic network loading model used for reference in this study employs a route structure in which route choice is allowed only in the departure (origin) zone. Once traffic is assigned to a route, the flow on this route can not be (partly) reassigned to another route.

Pipe concept

The reference DNL model MaDAM/Streamline uses pipes for the flow propagation. Pipes are created along a route, without connectors and splitters between the entrance and exit (origin and destination). The pipes are comparable to waterpipes: if you put something in, it will eventually come out, depending on the flow speed inside the pipe. Each pipe is a layer 'under' the network, where each layer identifies a single route and can also related to a specific user class or vehicle type.

Flow in the pipe is dependent on the conditions at the links in the top layer, the actual network. Density is equal for all pipes under the same link, but speed and flow may differ since alternate fundamental diagrams may be used for each layer. For instance heavy freight trucks will have a lower maximum speed on the highway than regular cars. For each time step in the dynamic network loading process, the density on all links is updated based on the in- and outflow (for all pipes).

3.4 Route choice concept

The framework is developed for application on large scale networks. Path searching while running the model will dramatically reduce model performance and is considered unnecessary if an adequate routeset is known beforehand. Therefore the routesets are to be derived before running the actual model. Discrete choice modelling is used to calculate probabilities for the routes in the routesets.

3.5 Interaction concept

There is no interaction between route choice and dynamic network loading while propagating. Route choice fractions are calculated before the propagation starts and are evaluated after the simulation. Multiple iterations are used for the purpose of convergence (reaching equilibrium). More information on this subject is presented in chapter 7.

4 Theory on route choice and routesets

This chapter describes the basic theories on routes. This includes both theoretical requirements on the knowledge of route choice as traveller behaviour and a global overview of routeset generation and filtering.

➥ *Paragraph 4.1 starts with exploring the characteristics of route choice. This is followed by a brief investigation of route choice factors in paragraph 4.2. Paragraph 4.3 is on theory of routeset generation, 4.4 on randomisation techniques and 4.5 on routeset filtering.*

4.1 Route choice basics

Bovy and Stern (1990) have investigated wayfinding and factors of route choice thoroughly. One of the basic fundamentals they state is that route choice is individual behaviour. For a macroscopic model we therefore assume route choice of a group is the result of many individual choices. Such an approach requires we first understand individual route choice behaviour. The next step is to determine how the choice of many individuals can be represented by choices made by a population.

4.1.1 Types of route choice

Based on observations, three types of route choice are defined (Bovy & Stern, 1990):

- Simultaneous choice
- Sequential choice
- Hierarchical choice

Before explaining what these types are, a definition is presented.

> ***Decision point***
> *A decision point is a node in a network where two routes of the set of alternatives for an origin and destination combinations split. The first decision point is the trip origin.*

Simultaneous choice

The traveller makes a choice for a route before making the trip. The choice set contains routes between the origin and destination of the trip.

Sequential choice

While travelling a decision maker faces decision points. When arriving at such a point, the traveller evaluates his choice by reviewing all routes from the decision point to the destination. He follows the best route to the next decision point. The links between two decision points form a subroute.

Hierarchical choice
The hierarchical choice is similar to the sequential choice except for the probabilities to choose an alternative. In the case of sequential decision making the choice of the next subroute is independent of previous choices, while in hierarchical decision making this probability is dependent.

In practice all three types of route choice occur (Jansen & Den Adel, 1987; Stern & Leiser, 1988; Benshoof, 1970; all in Bovy & Stern, 1990). However, even in simultaneous route choice it is likely that travellers can be forced to change routes because the route they did choose is no longer available, for instance because an incident has blocked a tunnel. This is referred to as adaptive route choice.

> ***Adaptive route choice***
> *When the traveller decides to change his (initial) route choice while travelling, based on changing circumstances he encounters, this is called adaptive route choice.*

The introduction of vehicle navigation systems have lead to an increase in adaptive route choice in recent years. Especially devices with real-time traffic information are capable of giving optimal routes from the current position of the traveller to the trip destination.

4.2 Route choice factors

As pointed out before, each traveller is a decision maker that makes an individual choice. The choice for a route is made based on the evaluation of the alternatives the individual faces. These alternatives form the routeset.

> ***Routeset***
> *A routeset S_n is a set of alternative routes as observed by individual decision maker n.*

The choice from the alternatives in the routeset is made based on route choice factors. These factors might be traveller-, trip- and route-specific attributes. Traveller specific attributes include age, sex, income level. Trip-specific attributes may include the purpose and travel mode. Route-specific attributes include route length, travel speed and number of traffic lights.

Examples

An ambulance driver will choose the alternative with the smallest and most reliable travel time and the least amount of speed bumps. A truck driver might be forced to use a specific route because his load contains specific materials that are only allowed on designated roads. And someone who travels to his work daily will probably always take the same route, independent of route characteristics, unless the route quality changes significantly.

Previous research has shown that route-specific attributes are the most important (Bovy & Stern, 1990, pp. 65-68; Fiorenzo-Catalano, 2007, pp. 110-112). An overview of route-specific choice factors for different traveller types as given by Fiorenzo-Catalano (2007) is summarized in table 1.1. Of course this list is incomplete. For more information see Bovy and Stern (1990, table 3.3, p. 68).

	Traveller type			
Factor	Road users	PT Users	Pedestrians	Cyclists
Access and Egress time		●		
Congestion delay	●			
Cost / Delay		●		
First leg's travel distance			●	
In-vehicle time		●		
Number of turns/curves	●		●	●
Number of transfers		●		
Pollution			●	●
Road quality/surface	●			●
Safety	●		●	●
Tolls	●			
Travel distance	●		●	●
Travel time	●		●	●
Waiting time at stop		●		
Waiting time for transfer		●		
Walking time for transfer		●		
Weather protection			●	●

Table 4.1 ***Main route choice factors for travel modes***
(adapted from Fiorenzo-Catalano, 2007, p. 111)

4.3 Routeset generation and filtering

Because each traveller is a decision maker, the ideal way to model route choice would be to know the routeset for each traveller and further know how they evaluate the alternative routes in their set. This requires knowledge of the routeset of an individual. This section describes ways to define the routeset.

It is important to understand that the route choice process is implicit and travellers can only to some extent explain their choice. The process of routeset generation is even more implicit. This means that we have to use model techniques to define the route choice set.

Fiorenzo-Catalano (2007) has develop a framework for the routeset generation process, including 4 basic steps in routeset generation:

- Step 1: Search a best route according to certain conditions;
- Step 2: Evaluate the route to a set of route criteria;
- Step 3: Select or reject the generated route;
- Step 4: Evaluate the resulting route set according to a set of criteria.

4.3.1 Types of routeset generation

In general there are three types of route generation:

- Single objective function search
- Multi-objective function search (Label search)
- Derive from capacityconstrained traffic assignment

The first and second approach use an objective function.

> ***Objective function***
> *The objective function is a function that represents the observed quality of a route by a decision maker. The function includes the attributes the traveller considers important (such as those described in table 4.1). The objective can be to minimise the function (e.g. the route length) or to maximise the function (e.g. route utility).*

The objective function is not limited to continuous values like link length and travel time, but can also be based on discrete values. For instance, a path search can be done to find the path with the smallest number of traffic lights.

4.3.2 Single objective function search

This generation type is based on a single, fixed objective function. The search for the shortest path in a network is an example of this type of routeset generation.

4.3.3 Label search

This type of routeset generation is based on multiple path searches, where the objective function is altered in each iteration. The weights for the specific attributes can be changed to let the objective function be more or less based on a specific attribute. A special variant is to use only one attribute in each iteration.

Example

An example of label search could be making a routeset with the shortest path, the fastest route, the route with the least number of speed bumps or the route as suggested by traffic signs.

Techniques for multiple routes from objective function(s)

For both techniques based on an objective function multiple routes for each objective function can be found by applying Monte Carlo simulation, where in each draw the attribute values – and thereby the objective function – take different values for each link. This results in other 'shortest paths'. An other technique is to eliminate one or more links in the path search process and thereby derive alternative 'shortest paths' which are added to the routeset.

> *For more information on techniques for the generation of multiple routes from objective function(s) see Fiorenzo-Catalano (2007), page 161 and further.*

Previous applications

Ben-Akiva *et al.* (1984) have proposed a labelling method using a large number of optimality criteria based on surveyed choice motivations. An optimal path is found for each of the criteria: shortest route, quickest route, best signposted route, scenic route, etc. Bekhor & Toledo (2005) state that six labels could cover about 90% of all travelled routes. Others suggest that approximately between 60 and 80 percent of the travelled routes can be identified (Ortúzar & Willumsen, 2001, pp. 328).

4.3.4 Derive routeset from capacity constraint traffic assignment

With capacity constrained assignment multiple path searches are done with the same objective function. The value of the link attributes are influenced each iteration by the assigned traffic rather than randomised. In the end, the routeset contains paths found as a shortest path in all iterations. Using this method, the routeset is dependent on the used assignment algorithm and its parameters (e.g. the fractions in incremental assignment or the number of iterations when using volume averaging techniques).

Example

Consider a road network with travel approaching supply. A single objective function path search might result in the least travel time, but the first iteration is based on zero flow on the network. In the second iteration, all traffic is assumed to use this route, which will increase the travel time and possibly will make another route faster. After several iterations the traffic is spread over multiple routes. Together these routes form the routeset.

4.3.5 Shortest path search

For the first and second type of routeset generation a shortest path search is used. This paragraph explains briefly what path search is and the elements of the basic algorithm.

> ***Shortest path search***
> *Shortest path searching is the process of finding the sequence of links in a weighted graph with minimal impedance.*

Path search algorithm

There is a wide variety of shortest path algorithms available. However, the basis of all these methods is the tree-building algorithm of Dijkstra (1959). This efficient node-by-node algorithm is very effective. For transport planning however, the presence of forbidden turn movements on intersections can not be modelled by the algorithm. A link-by-link algorithm has to be used.

4.4 Theory on randomisation

To find routes that are suboptimal either the shortest path must be ignored or have a larger impedance **or** a suboptimal path has to be have a lower impedance. A known technique for the first approach is used in the Marple traffic model, where repeated shortest path searches are performed and the impedances of the links of the found are multiplied by a factor. A drawback of this approach is that new found paths are unlikely to (partially) overlap with already found paths and therefore might lead to a unrealistic routeset.

Using randomisation techniques routesets can be generated that do not exhibit the drawback described above. Further they allow suboptimal paths to be found as optimal paths.

Randomising the link attributes is based on the fact that the link attributes are perceived differently among travellers. Examples of such attributes are travel time and travel distance. In the routeset generation process this behaviour is simulated.

General outline of Monte Carlo simulation

The Monte Carlo simulation technique exists of many shortest path searches on a transformed version of the network. The transformed state of the network is the result of randomising the attributes for a subset of links before the search. This subset might be either one single link, the links in the current routeset or the complete network.

Link attribute values are randomised using random number generators on a computer. Scaling problems might occur when absolute random numbers are generated: there is a large difference between the random outcome of 2 time units when travel time is measured in minutes or in hours. Relative random numbers do not exhibit this problem.

Distribution random function

The literature is not specific on what set of parameters and what distribution is to be chosen for optimal routeset generation. The choice for a set of parameters is quite arbitrary, but since routeset generation is mostly followed by a filtering process, the final routeset can be derived in many ways.

In this research the randomisation is assumed to be based on distribution of the link attributes, where each attribute might have a specific distribution. For many uncorrelated attributes the central limit theorem states that the link impedance is normally distributed.

One problem arises when using the Normal distribution. In the process of path searching it is required that no cyclic paths are found, and therefore all link weights should have the same sign. Using a Normal distribution with mean 1 (relative factor for link impedance) can however result in negative values. From the theory of attribute perception this is unlikely: a traveller might perceive an attribute value in a more positive way, but it is not likely to perceive this value having another sign.

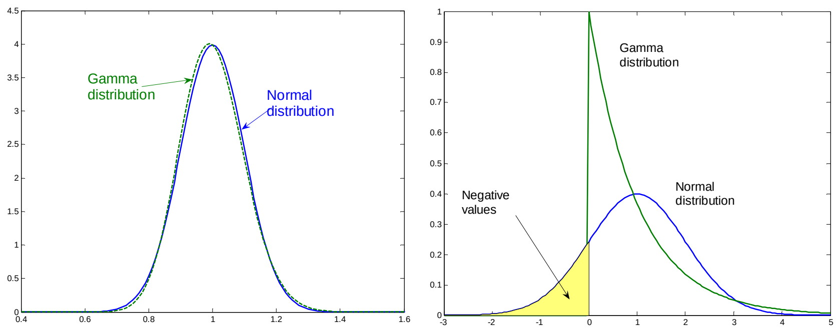


Figure 4.1 ***Gamma distribution and Normal distribution for μ=1.***
(Left figure σ^2 = 0.1, Right figure σ^2 = 1)

A solution for this problem can be found by using the Gamma distribution. Given a specific parameter combination this distribution is almost equal to the Normal distribution, but without negative values. For increasing variances the Gamma distribution results in positive skew, see figure 4.1.

Accelerated approach

The Accelerated Monte Carlo approach uses an initial low variance. When after some draws no new routes are found, the variance increases, leading to larger changes in the link impedances. The idea is that then routes are found further away from the shortest path.

4.5 Routeset filtering

Apart from the question of how routesets are generated, it is important to identify the specifications for adequate routes and routesets. Fiorenzo-Catalano (2007) presents a framework containing definitions of Dial (1971) and Hoogendoorn-Lanser (2005). This framework consists of requirements for single routes, comparing between routes and the total routeset and will be described briefly[3].

Requirements for single routes

- Reasonable routes are a-cyclic. All links have positive impedance values (Acyclic criterion).
- A reasonable route does not exhibit a detour from the shortest possible connection in terms of one or more measures such as distance or time between origin and destination larger than a maximum threshold α. (Detour criterion)
- A reasonable route is constituted by a systematic sequence of functional link levels, avoiding route parts going from higher to lower level links and back. (Hierarchical quality criterion)

Requirements for comparing alternative routes

- The mutual overlap between two alternative routes is less than Δ percent with respect to the shorter one of the two routes (Overlap criterion)
- Any two routes of the choice set should be comparable in travel (dis)utility within a given threshold of θ percent with respect to the shorter one of the two routes (Comparability criterion)
- The non-common parts of two partly overlapping routes should have a maximum detour not larger than a given maximum percentage ω_{max} of the minimum two parts (Detour-max criterion)
- The non-common parts of two partly overlapping routes should have a minimum detour not smaller than a given minimum percentage ω_{min} of the minimum two parts (Detour-min criterion)

[3] For more information on calculation and examples of the criteria see Fiorenzo-Catalano (2007, pp. 138-155).

Requirements for total route set

- All reasonable routes that are likely to be used are part of the routeset (Reasonable criterion)
- The size of the routeset is limited to a predefined number of S routes (Choice set size criterion)

The above mentioned requirements can be mathematically tested on the generated routeset and unfeasible routes can be filtered out to obtain a final choice set.

5 Route choice as discrete choice problem

In this chapter the route choice problem is described as a discrete choice problem. Several choice models are presented as possible methods to deal with the route choice problem. The models are described from the underlying theory of random utility maximisation. The descriptions are used to make a choice in the next chapter on what route choice model is preferred for application.

➥ *This chapter starts with an introduction in paragraph 5.1. Paragraph 5.2 gives an introduction to random utility maximisation theory. Basic discrete choice models are presented in 5.3. A problem arising when using these models is mentioned in 5.4 followed by exploring alternative discrete choice model formulations in paragraph 5.5. Finally, paragraph 5.6 presents a simulation technique.*

5.1 Introduction

In this chapter the following conditions are considered. A predefined routeset is used, containing a limited set of alternatives for an OD-pair. The modeller can specify utility functions which may include both fixed characteristics of the network (e.g. speed bumps) and measures of dynamic network performance. The choice model is supposed to result in the probabilities of a population of travellers choosing the specified route from the routeset.

5.2 Random Utility Maximisation theory

Route choice can be seen as a discrete choice problem. Random Utility Maximisation theory assumes that each traveller n will try to maximise his utility when making this choice.

Assume the routeset $S_n = \{R_1, R_2, ..., R_j, ..., R_J\}$ consisting of J alternatives. Each alternative known by decision maker n all have a personal utility U_{nj}, so the decision maker will choose alternative $R_j \in S_n$ for which $U_{ni} > U_{nj} \quad \forall j \neq i$.

For the researcher the utilities of the alternatives for an individual are unknown. However, to the researcher information is available of attributes x_{nj} that represent the alternative. Further attributes of the decision maker s_n are estimated, such as willingness to pay and sensitivity for route costs. The researcher defines the representative utility V_{nj} as a function that combines these attributes: $V_{nj} = f(x_{nj}, s_n)$.

Although it is theoretically possible that the researcher can define the exact utility, it is assumed that in general the representative utility approximates, but not equals the actual utility: $V_{nj} \neq U_{nj}$. Therefore $U_{nj} = V_{nj} + \varepsilon_{nj}$, where ε_{nj} captures the attributes that influence the utility of the decision maker, but are unknown to the researcher (Train, 2002). Bierlaire (2005) defines ε_{nj} as the capture of the maximum of many unobservable attributes and specification errors. Since $\{\varepsilon_{n1} \ldots \varepsilon_{nJ}\}$ are unknown, the set is assumed to be randomly distributed.

The probability a decision maker chooses alternative i can be rewritten in terms of the representative utility:

$$\begin{aligned} P_n(i) &= \mathrm{Prob}(U_{ni} > U_{nj} \forall j \neq i) \\ &= \mathrm{Prob}(V_{ni} + \varepsilon_{ni} > V_{nj} + \varepsilon_{nj} \forall j \neq i) \\ &= \mathrm{Prob}(\varepsilon_{nj} - \varepsilon_{ni} < V_{ni} - V_{nj} \forall j \neq i) \end{aligned} \tag{5.1}$$

Given the cumulative distribution of ε_n, the probability becomes

$$P_n(i) = \int_{\varepsilon} \mathrm{I}(\varepsilon_{nj} - \varepsilon_{ni} < V_{ni} - V_{nj} \forall j \neq i) f(\varepsilon_n) d\varepsilon \tag{5.2}$$

The probability is a multidimensional integral over the distribution of ε_n. The definition of $f(\varepsilon_n)$ defines the choice model. Only for certain specifications of $f(\varepsilon_n)$ a closed form solution of the multidimensional integral exists.

5.3 The Probit and Logit model families

In general, two families of choice models can be considered. Within the families variants may exist. The families are distinct since they have different base assumptions on the distribution of the unobserved portion of the utility function.

5.3.1 Multinomial Logit

The Logit model is derived under the assumption that ε_{ni} is distributed IID Extreme Value Type 1 for all i:

$$f(\varepsilon_{ni}) = \mu e^{-\mu(\varepsilon_{ni} - \eta_i)} e^{-e^{-\mu(\varepsilon_{ni} - \eta_i)}} \tag{5.3}$$

where μ is a scale parameter and η_i is the location parameter (alternative specific). Using the above definition for the distribution of $\{\varepsilon_{n1} \ldots \varepsilon_{nJ}\}$, the Multinomial Logit model can be derived (Ben-Akiva & Lerman, 1985, p. 106).

Multinomial Logit

$$P_{ni} = \frac{e^{\mu \cdot V_{ni}}}{\sum_{j \in C_n} e^{\mu \cdot V_{nj}}} \tag{5.4}$$

The probability function has a very convenient form, which makes it a popular model and easy to apply. However, the MNL model exhibits the IID-property: the unobserved factors are considered independent and identically distributed. This results in equal variances for all alternatives (Train, 2002). For application in route choice situations, this means the model does not take overlap and different variances among alternative routes into account.

5.3.2 Multinomial Probit

The Probit model is derived under the assumption that ε_{ni} is multivariate normally distributed with a vector of means $\overline{0}$ and a $J \times J$ variance-covariance matrix. The probability for an alternative can be written in RUM terms as

Multinomial Probit

$$P_{ni} = \int_{\varepsilon} \mathrm{I}(\varepsilon_{nj} - \varepsilon_{ni} < V_{ni} - V_{nj} \forall j \neq i) \phi(\varepsilon_n) d\varepsilon \tag{5.5}$$

where $\phi(\varepsilon_n)$ is the joint normal density with zero mean and the defined variance-covariance matrix. No closed form solution exists for the Probit Model. The probabilities are evaluated numerically through simulation (an algorithm to perform such a simulation is described in paragraph 4.6).

5.4 The overlap problem

In route choice modelling overlap between routes defines a correlation between the error terms and thereby influences the choice probabilities.

Overlap

Links (and corresponding nodes) present in two routes result in overlap. The route utilities are positively correlated with the amount of overlap.

An example of the overlap problem is presented in (Sheffi, 1985, p. 294). For effective route choice modelling, overlap should explicitly be taken into account in order to prevent wrong outcomes (Frejinger & Bierlaire, 2007). In the Probit model the variance-covariance matrix is explicitly defined. But since this method requires simulation it is far slower in performance than the Multinomial Logit model.

5.5 Alternative Logit formulations

During the last two decades alternative Logit formulations have been developed that capture the correlation among alternatives. A classification can be made on how these models take correlation into account.

5.5.1 Common links define nest structure

The MNL model does not contain levels. An extension is given by the Nested Logit model, where alternatives are placed in nests. Each nest contains correlated alternatives. Alternatives among different nests are uncorrelated. The bottom level nest is equal to a standard MNL model. Theoretically it is possible to define a model with a large number of levels (where each nest might contain subnests). See figure 4.1 for an example of a nested model for mode choice.

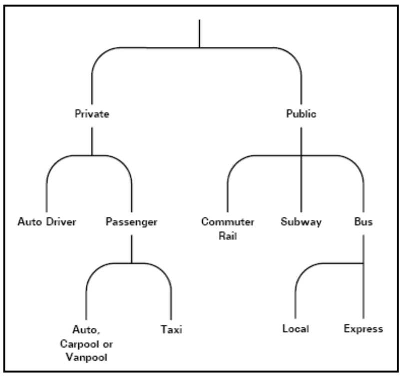


Figure 5.1 Example of nesting structure in mode choice
(From: Ramming, 2002, p. 37)

Cross-Nested Logit (CNL)
In the Cross-Nested Logit, the Nested Logit model is extended by a correlation parameter which makes it possible for an alternative to belong to several nests with different degrees. The degree of correlation is defined by a parameter α_{mi} $(0 \leq \alpha_{mi} \leq 1)$. This parameter was defined by Vovsha and Bekhor (1998) as the ratio of the common link length (or time) to the total in a route. For more

information see (Ramming, 2002, pp. 49-52). One important limitation pointed out by Fiorenzo-Catalano is that for realistic routes containing many links and larger routesets the nesting structure would be extra-ordinarily complex (2007, p. 126).

The probability is defined by

$$P_i = \frac{\sum_{m=1}^{M} \left(\alpha_{mi} e^{V_i}\right)^{\frac{1}{\mu_m}} \left(\sum_{j \in C_{mn}} \left(\alpha_{mj} e^{V_i}\right)^{\frac{1}{\mu_m}} \right)^{\mu_m - 1}}{\sum_{m=1}^{M} \left(\sum_{j \in C_{mn}} \left(\alpha_{mj} e^{V_i}\right)^{\frac{1}{\mu_m}} \right)^{\mu_m}} \tag{5.6}$$

with μ_m being the scaling parameter for the nest m.

General Nested Logit (GNL)

The General Nested Logit is an extended formulation of the CNL model. GNL allows an extra dissimilarity parameter between nests. In practice this would mean that links in different nests would contribute on different scales to the representative utility, which contradicts the concept of representative utility. This concept requires all links to be comparable, i.e. to have equal scales. According to Fiorenzo-Catalano (2007, p. 126) this makes GNL unusable for route choice modelling, although there might be some exceptional cases.

Paired Combinatorial Logit (PCL)

The Paired Combinatorial Logit uses a correlation parameter for each combination of routes (e.g. i and j) in the routeset. This correlation parameter is given by

$$\eta_{ij} = \frac{d_{ij}}{\sqrt{d_i \cdot d_j}} \qquad 0 \le \eta_{ij} \le 1 \tag{5.7}$$

where d_i is the length (or time) of route i and d_{ij} is the length of the common links. $\left(1 - \eta_{kh}\right)$ is a measure of the correlation between the alternative routes.

Because only each combination of alternatives has to be investigated, the structure of a PCL is less complex than the nested structure of CNL/GNL and thereby provide a computationally tractable mechanism for the route choice problem (Fiorenzo-Catalano, 2007, p. 126).

The probability in the PCL model is defined by $P_i = P(i \mid ij) \cdot P(ij)$ where

$$P_{(i|ij)} = \frac{e^{\left(\frac{\mu \cdot V_i}{1-\eta_{ij}}\right)}}{e^{\left(\frac{\mu \cdot V_i}{1-\eta_{ij}}\right)} + e^{\left(\frac{\mu \cdot V_i}{1-\eta_{ij}}\right)}} \tag{5.8a}$$

and

$$P_{ij} = \frac{\left(e^{\left(\frac{\mu \cdot V_i}{1-\eta_{ij}}\right)} + e^{\left(\frac{\mu \cdot V_j}{1-\eta_{ij}}\right)}\right)^{1-\eta_{ij}}}{\sum_{r=1}^{|S|-1} \sum_{p=r+1}^{|S|} \left(e^{\left(\frac{\mu \cdot V_r}{1-\eta_{rp}}\right)} + e^{\left(\frac{\mu \cdot V_p}{1-\eta_{rp}}\right)}\right)^{1-\eta_{rp}}} \tag{5.8b}$$

with scale parameter μ and $|S|$ defining the size of the choice set.

5.5.2 Common links define disutility component

The second type of alternative Logit formulations use a disutility component for accounting the overlap. The general idea behind this approach is that when making a choice between two partially overlapping routes, only the utility for the non-overlapping section is competitive and influences the decision. This approach is also known as penalising.

C-Logit (CL)

The C-Logit model penalises the alternative's utility function by a Commonality Factor (CF). This factor is defined as

$$CF_i = \gamma_0 \ln \sum_{R_i \in S_n} \left(\frac{d_{ij}}{\sqrt{d_i \cdot d_j}} \right)^{\gamma_1} \tag{5.9}$$

where γ_0 and γ_1 are positive parameters and have to be estimated (Fiorenzo-Catalano, 2007, p. 127). The difference in approach compared to PCL is that the factor is measured compared to all other alternatives in the set, while PCL makes a comparison for each combination of two alternatives. After application, testing and calibration, Ramming concludes that C-Logit does not give useful results for his case study in Boston for which route choice data was available (Ramming, 2002, p. 190).

Path Size Logit (PSL)

The Path Size Logit model uses a penalty for the Utility function, similar to the C-Logit model. The model is based on the Path Size-parameter PS, which is defined as (Generalised Path Size Logit)

$$PS_i = \sum_{a \in R_i} \frac{l_a}{d_i} \cdot \frac{1}{\sum_{R_j \in S_n} \left(\frac{d_i}{d_j} \right)^{\gamma} \cdot \delta_{aj}} \tag{5.10}$$

where $a \in R_i$ represents all links in route i, l_a is the length of link a, d_i represents the length of route i and δ_{aj} is a binary value that is equal to 1 if link a is present in alternative j and 0 otherwise. The parameter γ needs to be estimated (Hoogendoorn-Lanser *et al.*, 2004).

The probability is defined by

$$P_i = \frac{e^{\mu \cdot (V_i + \beta \ln(PS_i))}}{\sum_{R_j \in S} e^{\mu \cdot (V_j + \beta \ln(PS_j))}} \tag{5.11}$$

The Path Size Logit formulation is argued to sometimes give counterintuitive results (Bierlaire & Frejinger, 2005).

5.6 Probit simulation technique

5.6.1 Introduction

Although the Probit model can not be applied directly, a simulation technique can be used to derive choice probabilities. The simulation algorithm is described by Sheffi (1985). The basic principle is to randomise link cost values and thereby randomise the route utilities taking the overlap directly into account. When this is done for a large number of draws a multi-dimensional integration is simulated. Route probabilities are calculated from the percentage of draws a route is simulated as 'shortest path'.

5.6.2 Algorithm

For each OD-pair, consider a routeset $S^{OD} = \{R_1, R_2, ..., R_i, ..., R_J\}$. The links present in the routeset form the set $L = \{\ell_1, \ell_2, ..., \ell_a, ..., \ell_A\}$. The objective function uses travel cost and the aim is to minimise the function value. X is initialised as a zero-vector with J elements.

In each draw the link costs (or utilities) are randomised by applying

$$C^*(\ell_a) = (1 + \zeta_a) \cdot C^0(\ell_a) \forall \ell_a \in L \tag{5.12}$$

Because we are using the Probit model, we assume the random term ζ_a to be

$$\zeta_a \sim N(0, \sigma_a{}^2) \, \forall a \text{ [4]} \tag{5.13}$$

[4] In chapter 6 variance of routes is discussed.

After randomising the link travel cost, the link costs are summed over the links in a route to derive the random route travel cost, C*:

$$C^*(R_i) = \sum_{\ell_a \in R_i} C^*(\ell_a) \;\; \forall R_i \in S^{OD} \tag{5.14}$$

Now route costs are known for all routes in the routeset (for this draw). One of those routes has the lowest cost. This increases the number of draws for which the alternative is preferred.

$$X(i) \rightarrow \begin{cases} X(i)+1 & if\ C^*(R_i) = \min\limits_{R_j \in S^{OD}} (C^*(R_j)) \\ X(i) & otherwise \end{cases} \tag{5.15}$$

After all draws, the probabilities for each alternative are given by

$$P(i) = \frac{X(i)}{\#draws} \tag{5.16}$$

6 Route choice model analysis

The previous chapters have given an introduction to available route choice models. The purpose of this chapter is to analyse which route choice model is best to use for large scale application and therefore should be implemented in the framework. Thereto model performance is analysed using a case study.

➥ *This chapter starts with looking at relevant previous research in paragraph 6.1. In paragraph 6.2 the approach used for the performance analysis is presented. A case study is presented in paragraph 6.3. The analysis and test results can be found in paragraph 6.4 and discussed in paragraph 6.5. A remark on discrete choice models is made in paragraph 6.6. In paragraph 6.7 a proportionality factor is introduced, followed by a conclusion on which model is best to be used in paragraph 6.8.*

6.1 Previous research

All of the models in the previous chapter are well described (from a theoretical point of view) in scientific literature. The amount of publications on practical application of the models by others than those who defined them is however very low. Only two publications are worth mentioning. Ramming (2002) gives a thorough investigation of the models. He used the Path Size Logit model to calibrate data from a case study in Boston. Information on the quality of other models is not described.

More recent Bliemer and Bovy presented a paper in which they compared various Logit models using the Probit model. Their focus was on the prediction quality of the route choice models in dependence of the size and composition of pre-defined routesets (Bliemer & Bovy, 2008). They used a Probit simulation technique to determine fictive route choice probabilities and calibrated the Logit models against those. Then they changed the size of the choice set (adding or subtracting routes) to see how well the calibrated models approached the new calculated Probit simulation route choice probabilities. They found that none of the investigated models (MNL, C-Logit, CNL, PSL, PSCL and PCL) are robust and all models lead to incorrect probabilities after changing the size of the choice set. In their experiment Bliemer and Bovy used a simple 1 origin, 1 destination network containing 12 links.

6.2 Approach

The research from Bliemer and Bovy might indicate that Logit-based route choice models, despite intensive calibration, still lack robustness. The sensitivity to the routeset can be a problem for application in large scale networks, because in such networks the routesets are likely to have different sizes. Does this mean the route choice models are all useless or can we determine for what conditions the models give better or worse results?

This chapter investigates model performance of 5 Logit-based route choice models (MNL, CNL, PCL, PSL and C-Logit). The approach used is quite similar to the one used by Bliemer and Bovy. The simulation technique described in paragraph 5.6 is performed with a sufficiently large number of draws to derive route choice probabilities. The Logit-based models are then calibrated (at network level) against those probabilities. At level of routesets (per OD-pair) the model performance is then analysed against properties of the routeset.

6.2.1 Number of draws needed

To make realistic comparison between model performance, the number of draws used in the Probit simulation has to be large enough. Otherwise the stochastic spread in the probabilities from the simulation is too large. Therefore, the number of draws needed in the Probit simulation technique has been investigated.

Approach

In total 1.000.000 draws have been made, split up in batches of 100 draws. For each batch the route probabilities are calculated. By averaging over batches the number of draws increases and the probabilities converge to the average of all draws. For example, for the sixth batch the probabilities for the first six batches are averaged (assumed as 1 batch of 600 draws). The difference between the simulated probability for this collection of batches and the overall average probability is a measure for the robustness.

Mathematical formulation

Let P_i^b denote the route choice probability for route i from a certain routeset as simulated in batch b. The 'correct' probability is assumed to be the average of 1.000.000 draws, e.g. the average of the probabilities of all batches:

$$\overline{P_i} = \frac{\sum_{b=1}^{10000} P_i^b}{10000} \tag{6.1}$$

The average probability of a collection of batches is given by

$$\widetilde{P}_i^b = \frac{\sum_{q=1}^{b} P_i^q}{b} \tag{6.2}$$

The relative difference between the probability in a collection of batches and the overall average is given by

$$\frac{\left|\overline{P}_i - \widetilde{P}_i^b\right|}{\overline{P}_i} \tag{6.3}$$

This indicator for robustness can be calculated for multiple routes in a set and for multiple routesets, resulting in an indicator for robustness of the Probit simulation technique for varying simulation sizes.

Results

As expected the number of draws affects the robustness of the Probit simulation. A larger number of draws leads to a smaller relative error in the probability calculation. Figure 6.1 shows the relative absolute error (average over multiple routesets) from expression 6.3 for multiple simulation sizes.

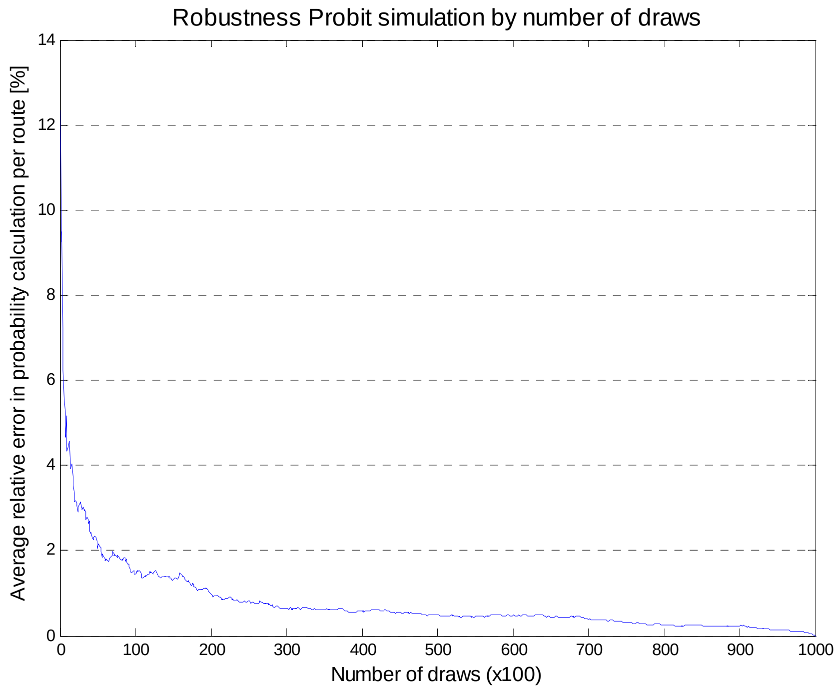


Figure 6.1 Robustness of Probit simulation

Discussion

It can be seen that up to 20,000 draws the robustness of the simulation increases significantly with the number of draws, while after 20,000 draws the increase in model performance is relatively small.

Ideally the number of draws would have to be set to a very large number. However, the computational burden of the Probit simulation technique is immense. Randomising a routeset consisting of 6 routes with 1000 draws takes between 0.5 and 1.5 seconds, depending on the number of links in the routeset. To compare: calculating probabilities for a routeset using a Logit-based model takes only a few microseconds.

Based on the results shown in figure 6.1 the number of draws needed for the purpose of this research can be set to 20,000. The relative error per route is about 1%, which is assumed low enough to compare the outcome with Logit-based models.

6.2.2 Definition of model quality

Per route the absolute difference between the probabilities from the simulation and Logit model is a measure for the quality of the model. By taking the square of this difference large errors are extra penalised.

Mathematical formulation

For a routeset S the probabilities for each route $R_i \in S$ are denoted by ${P_{im}}^S$ where m indicates the model/simulation. The performance of a model is given by

$$Q_m^S = \sum_{R_i \in S} \left(P_{i,m}^S - P_{i,probit}^S\right)^2 \qquad (6.4a)$$

for performance quality at routeset level. For overall performance a summation is made over routesets:

$$Q_m = \sum_S \sum_{R_i \in S} \left(P_{i,m}^S - P_{i,probit}^S\right)^2 . \qquad (6.4b)$$

6.3 Case study

Description of network

For the performance tests, a large scale Dutch main road network (Bereiksbaarheidkaart) is used. This network contains over 4000 zones and over 200.000 links. For 26 zones spread over the network a routeset is created and filtered[5]. The zones are both larger cities and more rural residential areas, distributed all over the Netherlands. The idea behind this approach is that routes with different lengths in both urban (more detailed network) and rural areas are selected, resulting in different type of routesets (length of route, number of routes in set and different type of overlap).

[5] The input parameters for the routeset generation and filtering are available in appendix B.

Routesets

Free flow travel time has been used for the generation of the routesets. A filter is used to reduce the sets to a maximum of 6 different routes and a maximum overlap of 60 percent. Further small detours and infeasible long detours have been filtered out. The resulting routesets contain a total of 2148 routes (average number of routes per OD pair 3.18).

Calibration

It is important to realise that no realistic route choice is simulated, but only a comparison between Probit simulation and different Logit-based models. Therefore, calibration of the Logit models is not done against empirical data, but against a arbitrary value for the only parameter in the Probit model.

The parameter for the Probit simulation is taken equal for all OD-pairs. For selections of OD-pairs, the parameters for the Logit model are calibrated. Initially all Logit models were calibrated using the same OD selection. If the validation showed unsuccessful calibration other (random) selections were generated (per Logit model), until all models resulted in the same error value or when no further enhancement of the model could be retrieved.

6.4 Results

6.4.1 Overall model performance

Figure 6.2 gives a first glance at model performance. Each diagram shows the sum of residuals at OD-level. The colour-scale is equal for all models, which means the darker an OD-square, the worse the model performs for the routeset of this OD-pair. Q_m is noted at the top of each diagram. Based on this first investigation one could conclude that C-Logit and Cross-Nested Logit perform better than the other route choice models. Further MNL seems to give wrong results. However, as can be seen the model performance is different among OD-pairs. This justifies a further analysis of model performance with discrimination to choice set properties.

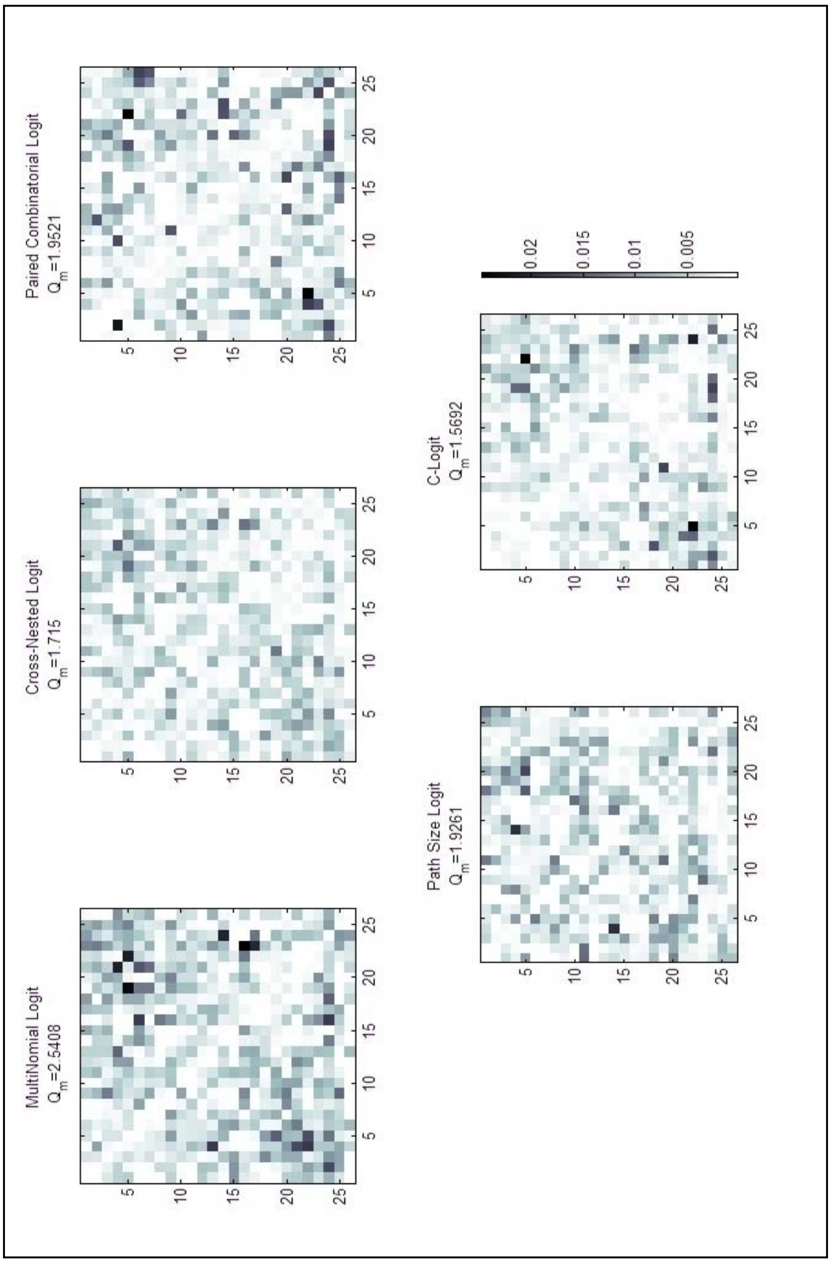


Figure 6.2 Comparison of route choice models for total route choice set

6.4.2 Size of choice set

A distinction of model performance can be made to the characteristics of the choice set. In this paragraph the size of the choice set is considered.

Choice set size	Model				
	MNL	CNL	PCL	PSL	CL
2	0.2774	0.1571	0.7076	0.3179	0.1204
3	0.6698	0.4514	0.3078	0.5634	0.3099
4	0.6317	0.4114	0.3518	0.5219	0.4289
5	0.3216	0.2435	0.1870	0.2221	0.2627
6	0.6403	0.4516	0.3980	0.3009	0.4474
Total	2.5408	1.7150	1.9522	1.9261	1.5692

Table 6.1 Model error by choice set size

Lower values mean better performance. The best performing model for each choice set size is coloured.

Table 6.1 shows the model error (sum of squares) for subsets of the zones used in the routeset generation. Each row in the table indicates a set of OD-pairs with a different number of alternative routes. Each column indicates a Logit variant model. Lower numbers mean better performance. Because the total error is different among the models, it is difficult to compare values between columns. The values can be compared between rows to see how well the same model performs on routesets with different sizes.

Results indicate that model performance changes with choice set size. For instance, the PCL model has a large error when predicting route choice probabilities for sets with only 2 alternatives, however for sets of size 3, 4 and 5 it performs best of all. The coloured cells indicate the best performing model per row. When all errors are scaled to a total error of 1.00 per model, this colouring holds and becomes even more significant.

6.4.3 Amount of overlap

Model performance is analysed with both average overlap and maximum overlap. For the case study routes have been filtered with more than 60 percent overlap. For overlap ranges between 0 and 60 percent the average[6] residual is calculated and plotted in figure 6.2. The figure shows that almost all of the OD-pairs with more than 1 route exhibit at least 2 routes with an overlap of more than 40 percent. The average overlap has a wider distribution among the OD-pairs. Especially this property has influence on the model performance.

[6] Since the number of OD-pairs exhibiting a certain overlap range varies among overlap ranges, the average residual error has to be calculated to obtain comparable diagrams.

For routesets with small overlap ranges (between 0 and 0.2) MNL and PCL perform well, while the other models (especially PSL) give wrong results. For the middle range (20 to 40 percent overlap) both PCL and C-Logit perform well, although the relative difference with CNL is small. For routesets with high amounts of overlap (40 to 60 percent) PSL outperforms all other models. CNL and C-Logit are comparable. Remarkable is the worse performance of PCL for overlap categories between 52 and 60 percent, which indicates this model might not be useful for routesets in which the overlap among routes is very high.

6.4.4 Length of route

The third and final analysis considers the cost of a route. In general longer routes have larger costs. A short analysis is made of the model performance per cost category. As depicted in figure 6.3, there is no real difference in model performance for the most common route costs. For high, rare route costs, PSL seems to perform slightly better, but no significant difference can be measured. However, PSL seems more robust compared to the other models, since performance is not dropping when the model is applied to routesets with route costs different from those used in calibration.

6.5 Discussion

When comparing the probabilities of the Logit models with the Probit model outcomes, three properties of the routeset have been analysed. The size of the routeset gave a first indication on model performance. A second analysis showed an even stronger relationship between routeset properties and model performance: some models perform well when the overlap among routes is small, while others perform better with higher overlap ratios. A third analysis on the route cost did not show such a strong distinction. However, it showed that the PSL model is more robust than the other models.

No single model performs best for all types of routesets. An ideal strategy would be to automatically determine the model to use for a specific routeset. If the application of a single model for all routesets is preferred, CNL, PCL or C-Logit is advised. PSL is not preferred for wide application, since the performance for short-length and barely overlapping routes is poor.

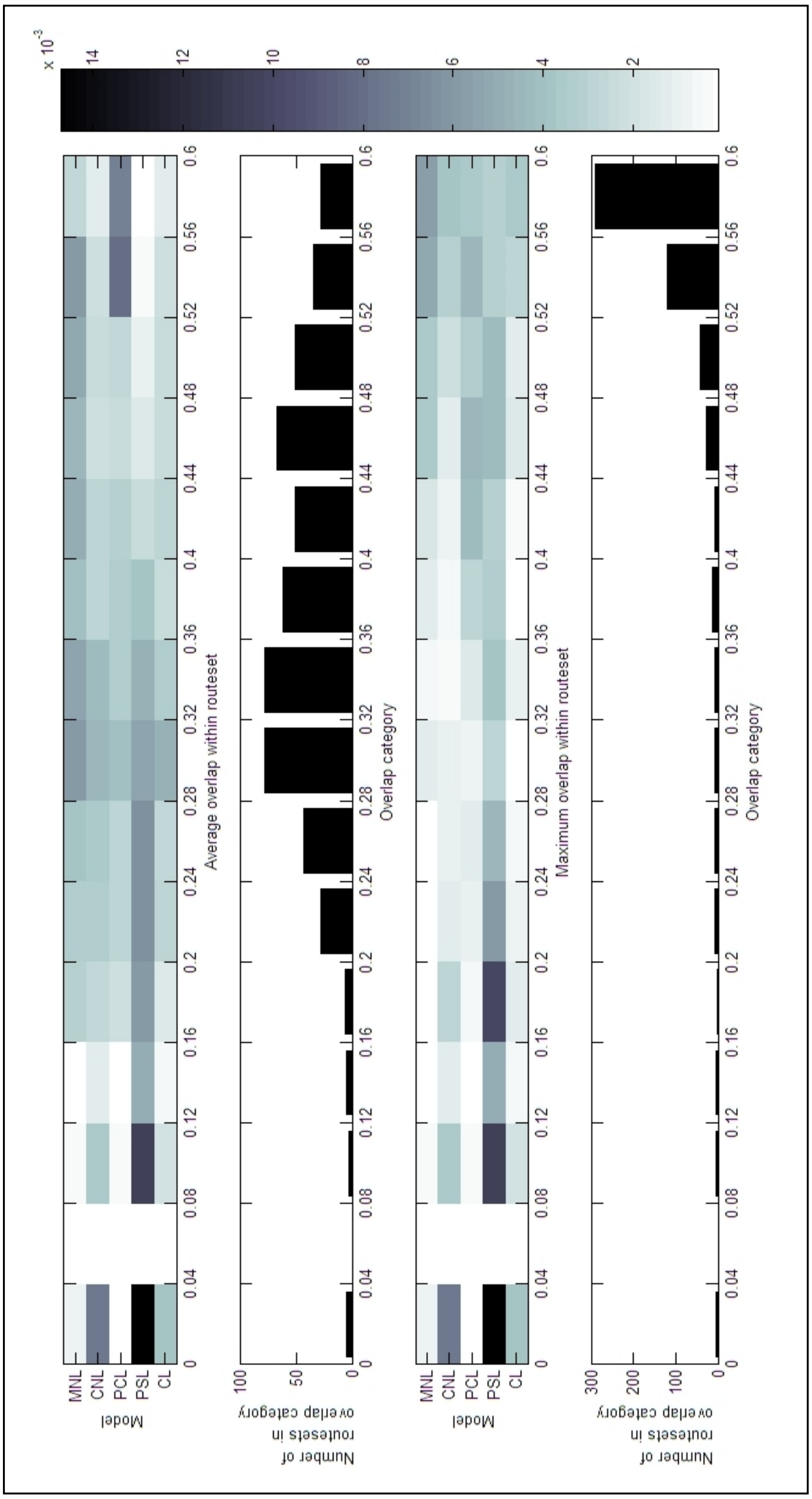


Figure 6.3 Model performance per overlap category

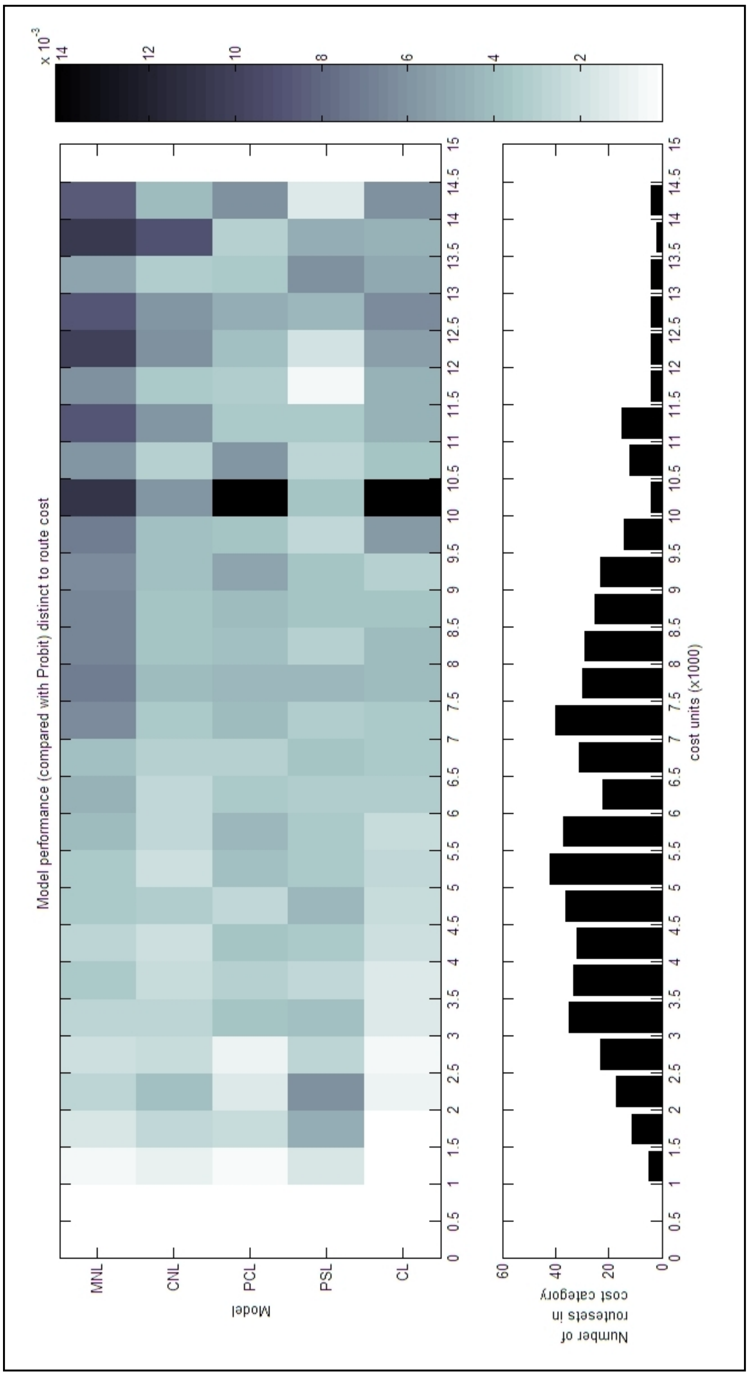


Figure 6.4 Model performance per route cost category[7]

[7] There large error in the cost category 10,000 to 10,500 seems to be the result of a specific routeset in this category. Because of the low number of sets in the category, the outlier dominates the average error.

6.6 Shortcomings of discrete choice models

The basic discrete choice model structure is not sufficiently qualified for large scale application. This is the result of two shortcomings. First, each of the route choice models described in chapter 5 requires at least one parameter to be estimated, the scaling parameter. Secondly the definition of variance is required to be more consistent with traveller behaviour.

6.6.1 Scaling

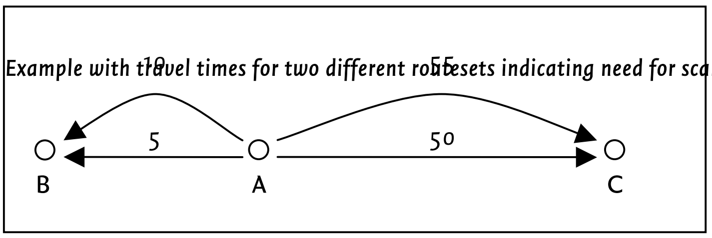
As shown in chapter 5, all discrete choice models use a scale parameter μ. The theoretical basis for the use of this parameter comes from the formulation of $\{\varepsilon_{n1} \ldots \varepsilon_{nj}\}$. Since the unobserved components are distributed Extreme Value type 1, the *default* variance of a random value ε_{ni} equals $\pi^2/6$. This variance has to relate to the value of utility and represents the behaviour of travellers to choose a route other than the utility maximising route.

Depending on the scale of utility, the ratio $\pi^2/6\mu^2$ to utility can be either small, reasonable or large, with results varying respectively from choosing the optimal route only, choosing between reasonable routes and ignoring the utility values/ choose randomly.
Since utility can be measured in any dimension, a correction has to be applied to rescale the utilities to match the variance of the unobserved terms.

The purpose of rescaling can be illustrated with the following example.

Consider a simple network with three centroids (A,B and C). There are in total 4 routes, all originating from centroid A. Two routes end in centroid B and have respective travel times of 5 and 10 minutes. The two other routes end in centroid C and have travel times of 50 and 55 minutes respectively. Lets consider travel time as only component of the disutility-function. Now, without rescaling, the probabilities will be equal for both routesets, since in both cases the absolute difference in utility equals 5. However, for destination B the second route takes double travel time relative to the first, while for destination C the additional travel time of the slower route is only 10 percent of the travel time of the fastest route.

Figure 6.5 Example with travel times for two different routesets indicating need for scaling



In the ideal case, each routeset has an individual scaling parameter, calibrated from empirical route choice data. However, for large scale application this is very costly and practically undoable. A practical workaround would be to use a limited amount of representative estimates as scale parameter for routesets with similar characteristics. From a behavioural perspective another approach can be derived. The next paragraph will focus on this.

6.6.2 Variance

Apart from the need for different relative variance scales for different routesets, an additional problem exists. The basic Logit formulation assumes all routes in a set to have equal variance. However, from behaviour, it can be argued that utility might be perceived differently among routes in the same routeset.

Following Daganzo & Sheffi (1977), (co)variances are assumed to increase with (common) route lengths. Or, more generally, the variance of route costs is positively related to the route cost itself. The following model is mentioned in the literature (Bovy, 1990).

$$\sigma_R^2 = \theta \cdot (\eta_R)^m \tag{6.5}$$

where θ and m are positive constants and η_R denotes the cost of route R.

Possible values of m include ½, 1 and 2. As stated in Bovy (1990, pp.74-75), only for $m = 1$ the variance at route level is consistent with the variances at link level: a summation of the variances for links in the route equals the variance of the route. However, this directly implies impedances of sequential links to be fully independent. For application this is an advantage, because cross-correlation does not have to be considered.

Effect of $m = 1$ on network loading

From behaviour one could argue that travellers generally do not perceive utility at single link level, but for series of links in a route (a subroute). An advanced version of the model could include an intelligent algorithm that identifies subroutes and base route choice on those subroutes. For now, $m = 1$ is considered good enough for the purpose of this research.

6.7 Variance scaling with a proportionality factor

The variance at route level of the MNL model is by definition

$$\sigma_R{}^2 = \frac{\pi^2}{6\mu^2} \tag{6.6}$$

Since the variance to mean ratio holds at both link and route level, the route variance can be replaced which leads to

$$\sigma_R{}^2 = \frac{\pi^2}{6\mu^2} = \theta \cdot \eta_R \tag{6.7}$$

hence

$$\mu^2 = \frac{\pi^2}{6\theta\eta_R} \Rightarrow \mu = \frac{\pi}{\sqrt{6\theta\eta_R}}\,. \tag{6.8}$$

For application μ has to be constant in a routeset. For routesets containing more than one route, μ is based on the minimal route cost.

In this form a direct relationship can be seen between the scaling parameter μ and the travel cost. The only unknown parameter is a proportionality factor θ. Instead of calibrating each routeset, now the proportionality factor has to be calibrated. At first glance this might not seem to be an improvement. However, since θ is derived from travel cost variance the parameter can be linked to user classes, trip characteristics and link type. Further, the proportionality factor can be influenced in the model based on the level of information of travellers.

This new approach is considered significantly more flexible for application and easier to apply to characteristics of model elements (network links, user classes, dynamic measures, etc.). However, additional research is needed to see to what extent this new method can be implemented for large scale modelling and to what extent it performs better than a scale parameter for each routeset. Elements that should be covered in that research are how multiple proportionality factors can be used to determine the scale parameter for one routeset.

6.8 Conclusion

The purpose of this chapter was to analyse which route choice model is best to use for large scale application and therefore should be implemented in the framework. A case study on a real network has shown that although calibration at individual route sets is needed, use of route choice models calibrated for a large collection of routesets does not lead to very bad results. Especially when characteristics of choice sets are taken into consideration. It was shown that size of routeset, amount of overlap and order of magnitude are all affecting the performance of each of the route choice models. When one model has to be chosen, it should be PCL. Although not the best model overall, this model is easy to estimate (only 1 parameter) and gives adequate results for most situations.

7 Dynamic network loading with route choice

The previous chapters have focussed on some of the individual elements in the framework. This chapter elaborates on the theory behind the interaction between DNL and route choice. Further it determines what type of interaction fits best for large scale application.

➥ *First, paragraph 7.1 describes the interaction between route choice and dynamic network loading roughly. Dynamic equilibria are investigated in paragraph 7.2, followed by elaborating on the assignment algorithms in paragraphs 7.3 and 7.4. A new approach is presented in paragraph 7.5. The corresponding mathematical model is described in paragraph 7.6 and the solution scheme is presented in paragraph 7.8.*

7.1 General concept of network loading and route choice

A numerical approach is used to make the framework fully flexible to function with any dynamic network loading model. This approach is iterative and requires multiple assignments with alternate inflow patterns before equilibrium is reached. These inflow patterns are derived from judging the outcome of previous iterations (for the first iteration other methods are needed).

The utility functions used for the choice modelling are evaluated after each iteration. For static assignment the function values are fixed. For dynamic assignment however, the function values are dynamic if they include dynamic elements such as travel time. More specific, the function value is a path integral of the function elements. Evaluated utility functions are used for the route choice model. Iterations are needed until the inflow pattern reaches a stable state.

7.2 Dynamic equilibrium definitions

In paragraph 2.3 the use of equilibria was introduced for traffic modelling in general. This paragraph focuses on dynamic equilibria.

7.2.1 Boston Traffic Equilibrium

The Boston traffic equilibrium is a dynamic generalisation of Wardrop's user equilibrium. It is based on the assumption that travellers try to optimize their routes based on network conditions at their time of departure. A flow pattern is said to be a Boston equilibrium when for each instant in time, for each OD-pair, the flow unit costs on utilised paths are equal to the minimum instantaneous unit path cost (Friesz *et al.*, 1993). This equilibrium is also known as the dynamic user optimal assignment (Ran *et al.*, 1993, Kuwahara & Akamatsu, 2001) and reactive/naïve route choice equilibrium (Han, 2000).

7.2.2 Simultaneous Route-Departure Equilibrium

When travellers for each origin-destination pair have equal travel cost (including time penalties for early or late arrival) regardless of route choice, the flow pattern is said to be a simultaneous route-departure equilibrium (Friesz *et al.*, 1993).[8]

7.2.3 Dynamic User Equilibrium

At equilibrium, for each origin-destination pair and for each departure time instant, the actual flow costs from time of departure to time of arrival on utilised paths are identical and equal to the minimum unit path costs which can be realised from among all route choice decisions (Bliemer, 2001). This definition is similar to the previous definition (7.2.2) except the left-out of departure time choice and is also known as predictive and realistic route choice (Han, 2000).

7.2.4 Deterministic versus stochastic equilibria

Each of the described equilibria can be deterministic or stochastic. Equal to the static case, a deterministic equilibrium is based on the assumption that all network users have perfect information of network conditions and determine their routes without errors. Stochastic models relax this assumption and assume that travellers will not make perfect decisions due to perception errors or based on attributes that are not part of the objective function the modeller uses.

When a discrete choice model is used, stochastic effects are introduced automatically (since the choice model contains unobserved/error terms). The variance of route costs, in the model influenced by the scaling factor μ, determines how well the stochastic equilibrium approaches the deterministic equilibrium.

[8] It should be noted that departure time choice is beyond the scope of this research

7.3 Characteristics of algorithms

Models that combine route choice and dynamic network loading have two main characteristics that identify the model: the type of equilibrium aimed for and the complexity of the dynamic network loading model. The functional algorithm has two characteristics that are derived from the model configuration: the number of runs and the run direction. All of these items will be briefly discussed below.

7.3.1 Equilibrium definition

The equilibrium definition used in the algorithm determines for a large part the two characteristics just mentioned. Models based on instantaneous travel cost do not need to re-evaluate route choice based on actual travel costs. Such models therefore can be single-directed, single-run methods. Models based on actual travel cost need multiple runs.

7.3.2 Complexity of the dynamic network loading model

Analytical formulation

When the dynamic network loading model uses simple equations, it is possible to present the assignment problem as an analytical problem. Then existing mathematical methods can be applied to derive equilibrium flow patterns.

Four mathematical formulations are known: the non-linear programming problem, the optimal control problem, the variational inequality problem and the complementarity problem. For a chronological overview see table 2.1 in (Bliemer, 2001, p. 16). As reported by Bliemer, there is a tendency to variational inequality formulations.

Iterative formulation

For models with more complex propagation functions, the mathematical formulation of the model becomes so complex, that it is not directly solvable. Instead, an iterative approach is needed. This means that at least multiple runs or multiple directions are used to derive the flow pattern.

7.3.3 Number of runs

A run is defined as running the model for the full simulation time. Algorithms can be single-run methods and multi-run methods. Multiple runs can be used for the purpose of iterating: using data from the previous run to enhance the model result. Another reason can be to have multiple runs, with different purposes for each run.

7.3.4 Run direction

Apart from the number of runs, models can be single-directed and multi-directed. In multi-directed models the model can go back in time, for example when a grid lock is found. Then the model can change strategy and run clockwise again.

7.3.5 Convergence to equilibrium

Due to the complexity of the dynamic network loading model and the route choice application, it is not guaranteed that an equilibrium can always be found using iterative methods. It is important to find a good start position from where the equilibrium is searched.

The main difficulty is the time aspect in the propagation: when in one run/iteration a 'wrong' decision is made, this must be corrected in later iterations. Depending on the specification of the model, this might not always be possible. Heuristic methods that are able to go back in time decrease the chance of definitely going a wrong way, because they evaluate the decisions while applying them.

7.4 Existing algorithms

Several existing dynamic traffic assignment algorithms have been described by (SWOV, 2003). Four main types of dynamic algorithms are distinguished:

- Multiple assignment algorithm
- Time dependent Frank Wolfe algorithm
- Subpopulation feedback assignment
- Individual feedback assignment

The last method is only usable for microscopic models. The applicability of the other methods is briefly discussed below.

Multiple assignment method

The simulation tools INTEGRATION and CONTRAM both use a two-phase model. First a pre-run is performed based on instantaneous network condition (i.e. no forecasting). The second phase iterates using the conditions from the previous iteration to redefine the spread of traffic over the routes.

This method is successful, but requires multiple iterations to converge. It is possible this method leads to oscillating. For large scale models this method might be sub-optimal.

Dynamic Frank Wolfe assignment

The Frank Wolfe algorithm, known from static assignment, can be 'dynamised'. Assignment is performed for many small time slices. The method uses iterations, where each iteration is used to perform a shortest path search. When new routes are found the routeset is expanded. The method uses successive averages (or – if possible – a line search technique) to combine multiple iterations and derive probabilities. This approach does not support the use of a route choice model and is therefore unsuitable for the purpose of this research.

Subpopulation Feedback Assignment

The mechanism of Subpopulation Feedback Assignment (SFA) updates the route choice of only a part of the population each iteration. Thereby it prevents oscillating effects. Multiple iterations are required to get the same effect as the FW-method. However, SFA is more elegant. In early versions of the micro-simulation tool INTEGRATION this method was used by default.

7.5 The dynamic forecasting approach

7.5.1 Introduction

In paragraphs 7.2 and 7.3 the concept of realistic and naïve route choice was discussed. Apart from the stochastic character of a discrete route choice model, there is reason to doubt both realistic and naïve route choice.

Firstly, there are situations in which a full dynamic user equilibrium is unrealistic. For instance, during off peak hours more irregular trips are made, for which drivers have no idea on future network conditions. Another example include situations in which special dynamic traffic management measures are taken during the trip. From a behavioural point of view: people can not know that these measures are applied when they plan their trip, so the model should not take these measures into account when determining the optimal flow pattern.

Complementary, the opposite method using instantaneous route choice, is considered false too. It is highly unrealistic to assume that travellers have no single idea on future network conditions, while their knowledge on current conditions is assumed perfect. Travellers always have some amount of information on the network at the moment of their departure. Based on their experience and network knowledge they can – up to a certain degree – estimate the short term future network conditions. Of course this degree vary among drivers.

The literature does not present a way to model this intermediate travel behaviour. Only the full DUE and Boston equilibrium are mentioned. This paragraph will elaborate on a new approach which makes it able to model travellers that behave with varying forecasting knowledge, fully compatible with the two mentioned equilibria.

7.5.2 Definition of equilibrium

An intermediate equilibrium is derived from the behavioural theory that travellers choose a route based on their expectations of future network conditions. Future network conditions can be estimated only limitedly. Route choice therefore depends on a dynamic estimation of travel cost for a part of the trip and a instantaneous estimation of travel cost for the rest of the trip.

> ***Definition***
>
> *The traffic is said to be in n equilibrium state when travellers on a specific OD-relation have equal total travel costs, where travel costs consist of path integral costs from departure to some point along a route and instantaneous travel costs for the part of the route from this point to the destination. The intermediate point is defined by the extent to which a traveller can predict future network conditions. This point can vary with user class.*

7.6 Mathematical formulation

In the following the travel costs exist for discrete time and discrete space. Costs are denoted by $c_{x,t}$ where x defines space and t defines time. Note that x and t are directly related by the route and propagation.

Route costs are denoted by $c^{rod}(k)$ for travellers using route r departing from origin o to destination d during time interval k. λ defines the horizon for which a traveller can predict network performance.

Route costs are calculated as follows.

$$c^{rod}(k) = \underbrace{\sum_{\tilde{t}=t_k}^{t_k+\lambda} c_{x^{rod}(\tilde{t}),\tilde{t}}}_{\text{A}} + \underbrace{\sum_{\tilde{t}=t_k+\lambda}^{t_k^{\infty}} c_{x^{rod}(\tilde{t}),t_k+\lambda}}_{\text{B}} \qquad (7.1a)$$

Alternatively this can be formulated as

$$c^{rod}(k) = \underbrace{\sum_{\tilde{x}=x^{rod}(t_k)}^{x^{rod}(t_k+\lambda)} c_{\tilde{x},t(\tilde{x})}}_{A} + \underbrace{\sum_{\tilde{x}=x^{rod}(t_k+\lambda)}^{x^{rod}(t_k^{\infty})} c_{\tilde{x},t_k+\lambda}}_{B} \tag{7.1b}$$

Where

A = Actual travel costs (path integral) for part of route travelled from origin up to the time horizon.

B = Expected travel costs (instantaneous) for rest part of route, based on network conditions at time horizon.

Minimum total travel cost at OD-level is denoted by $\pi^{od}(k)$.

$$\pi^{od}(k) \equiv \min_{r \in C^{od}} c^{rod}(k) \;\forall o,d,k \tag{7.2}$$

Deterministically, the network is said to be in an equilibrium state if

$$\left[f^{rod}(k) > 0 \Rightarrow c^{rod}(k) = \pi^{od}(k)\right] \;\forall o,d,r \in C^{od},k \tag{7.3a}$$

and

$$\left[f^{rod}(k) = 0 \Rightarrow c^{rod}(k) > \pi^{od}(k)\right] \;\forall o,d,r \in C^{od},k \tag{7.3b}$$

where $f^{rod}(k)$ denotes the flow on route r from origin o to destination d departing at time instant k.

Note that for stochastic route choice models this state can only be approached up to a certain degree.

7.7 Solution scheme

7.7.1 General outline

A double iterative approach is used to derive the equilibrium flow pattern. Figure 7.1 schematically shows this approach. The example figure considers a total simulation period of 90 minutes. Intervals span 10 minutes each. Route choice is constant for travellers for a specific OD-pair departing during the same interval, but might differ among intervals. Each interval therefore has a specific set of route choice probabilities for all routes.

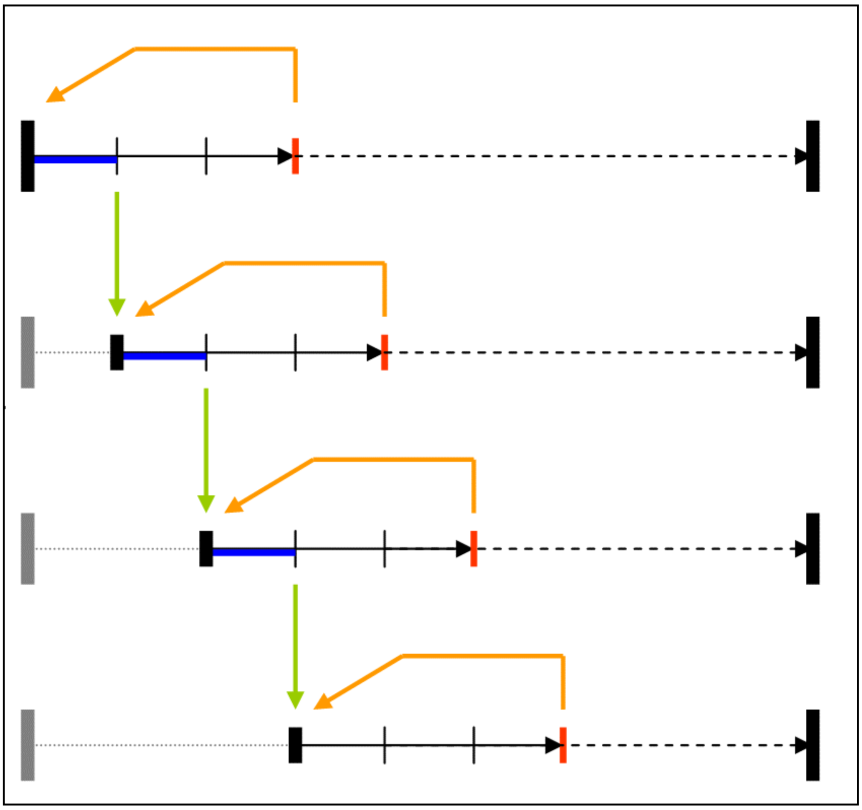


Figure 7.1 Outline of iterative process to solve assignment problem with forecasting

The model uses intermediate points (thick black bar; at the beginning of new intervals) for which traffic in the network is stored. For the travellers entering the network in the upcoming interval flows are calculated based on ‘expected’ travel conditions. Flows are added to the network and the DNL model is ran for the period up to the horizon (red bar; in the example of figure 7.1 assumed to be 30 minutes). After the time horizon minutes the route fractions are recalculated based on the actual network conditions. This process is repeated several times (orange arrows) until the route fractions converge. Then the last fractions are used to model up to the start of the next interval (blue line) where the process starts again (green arrow).

7.7.2 Main loop description

The central element is the dynamic network loading model. Traffic flows through the network using the MaDAM propagation model. At the beginning of each route choice interval k the route choice model is called to calculate the route choice probabilities, which are multiplied by the travel demand to determine inflow.

First, the costs from the previous run (or initial costs) are used to determine a new set of probabilities, only related to the current iteration and departure interval.

$$P_{rk}^{*od}(i) = f\left(c^{rod}(k, i-1)\right) \quad \forall r, o, d, k \tag{7.4}$$

Then the probabilities are weighted with the probabilities of the previous iteration.

$$P_{rk}^{od}(i) = (1-\alpha_i)\cdot P_{rk}^{od}(i-1) + \alpha_i \cdot P_{rk}^{*od}(i) \quad \forall r, o, d, k \tag{7.5}$$

Finally, the flow can be calculated using

$$f^{rod}(k,i) = P_{rk}^{od}(i)\cdot q_k^{od} \quad \forall r, o, d, k\,. \tag{7.6}$$

The weighting factor $0 \le \alpha_i \le 1$ is unknown. Optimal values for α_i are unknown, so α_i is to be estimated. One common approach would be to use successive averages, where α_i is defined by

$$\alpha_i = \frac{1}{i}\,. \tag{7.7}$$

Stop criterion

A duality gap is used to check convergence of the assignment. It is defined at time instant level by

$$DG = \frac{\sum_k \sum_{od} \sum_r \left[c^{rod}(k) - \pi^{od}(k)\right]\cdot f^{rod}(k)}{\sum_k \sum_{od} \sum_r \pi^{od}(k)\cdot f^{rod}(k)}\,. \tag{7.8}$$

For non-equilibrium results DG will be a positive number indicating the assignment error. In the equilibrium state, all used routes costs are equal to the minimal route costs, so the nominator will equal zero $\forall r, o, d, k$ and therefore DG will be zero.

DG is calculated after each iteration. Reaching a full equilibrium state is not always possible. Therefore, convergence of DG is used as a stop criterion. The algorithm stops if a sequence of iterations result in (approximately) equal values of DG (e.g. 3 successive iterations).

7.7.3 Inner loop description

The orange loop from figure 7.1 is referred to as the 'inner loop'. This paragraph gives a short outline of this model element.

Idea

The idea behind the inner loop is to react better on traffic conditions occurring in the near future. Traffic departing at t_k^0 (for a certain time instant k) make route choice decisions only based on travel costs during t_k^0 and t_k^λ. In case the horizon is small relative to the total trip distance, it could be more effective to use additional simulations for short term predictions, with the intention to reduce the number of full simulations and thereby reduce model run time.

Approach

The main loop calls the route choice model at the start of every new instant k. The route choice model then stores traffic conditions for t_k^0, calculates probabilities based on the previous iteration (similar to the way it is done in the main loop). Then traffic is loaded to the network and the propagation continues to t_k^λ, however, with a propagation model that not necessarily has to be the same as the model used for propagation in the main loop. For instance, a large time step can be used or even a completely different propagation model.

Stop criterion

At t_k^λ the inflow $f^{rod}(k)$ is evaluated using a time instant duality gap $DG(k)$

$$DG(k) = \frac{\sum_{od}\sum_{r}\left[c^{rod}(k) - \pi^{od}(k)\right]\cdot f^{rod}(k)}{\sum_{od}\sum_{r}\pi^{od}(k)\cdot f^{rod}(k)}. \tag{7.8}$$

Comparable to the main loop $DG(k)$ is checked for convergence. If $DG(k)$ is decreasing, the inflow is calculated again, this time using the output of the 'fast simulation' as input for the dynamic route cost calculation and the process is repeated. When $DG(k)$ is converged the main simulation is continued with the last calculated inflow for time instant k.

Pros and cons of the inner loop

The runtime of a single full model run increases when the inner loop method is used. This is the result of the additional inner loops and the need for extra data handling. When a less complex propagation is used, the additional simulation time can be limited. Depending on the complexity of the route choice problem, it might be possible to find a solution faster using the inner loop method. This, however can not be proven mathematically.

7.7.4 Determine route pattern for first iteration

The route costs $c^{rod}(k)$ are derived from the previous iteration. For the first iteration no dynamic costs can be calculated. Instead a static (stochastic equilibrium) assignment can be used to make a rough estimate of link flows. Other possibilities are using free flow conditions to predict travel costs for the route choice calculation or to use uniform probabilities.

7.8 Conclusion

The literature only proposes route choice behaviour in two ways, of which both are considered unsuitable for all situations. More in particular an intermediate method is currently missing. Such a method is introduced, including a mathematical description. Also a solution scheme is presented that includes an inner loop structure for better estimating behavioural effects.

8 Case study

8.1 Outline

In the previous chapter method for combining route choice and dynamic network loading was presented. This chapter tries to test this method by using a simple case study for application of the method.

8.1.1 Description

The new approach described in the previous chapter is tested using a case study. A 25-zone network model (Zuid-Meerendal) is used with realistic travel demand.

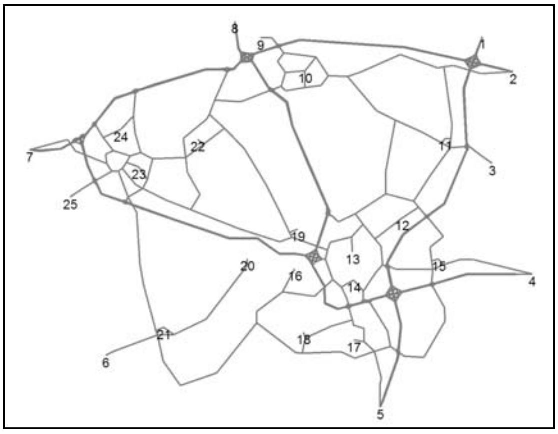


Figure 8.1 Network used for case study

A 30 minute period is modelled with route choice intervals of 5 minutes. The aim of the case study is to optimise route choice for the interval ranging from 10 to 15 minutes based on a 20 minutes horizon. The first two time intervals use uniform route choice (all routes for an OD-pair get assigned the same amount of demand), to represent non-optimal network conditions. The third interval uses the inner loop method described in the previous chapter. The 3 remaining intervals base route choice only on instantaneous travel cost.

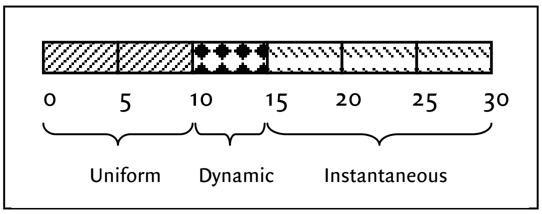


Figure 8.2 Types of route choice in the case study

8.1.2 Expectations

Because the first and second interval use uniform route choice, early departs result in a fixed load on the network. Traffic departing during the objective interval will optimise their route choice subject to this fixed load and subject to interacting traffic departing in later intervals. The objective interval traffic will approach an equilibrium.

8.2 Approach

A prototype model framework including the route choice, route cost calculation and flow propagation was built in Ruby and implemented in the OmniTRANS model environment. Route choice is performed with the PCL model.

8.2.1 Scenarios

The model takes a long time to run: 1 iteration including route cost calculation, route choice calculation and propagation of traffic takes about 45 minutes. Because of limited time, only a few scenarios have been used for model testing.

Scenario dimensions

Each scenario is based on four dimensions:

- Spread parameter μ
- Calculation of α between iterations
- Number of iterations
- Definition of initial route probabilities

Spread parameter

The parameter used for the spread in the Probit simulation model has to be chosen in such a way that it suits the order of magnitude of the cost attribute. Because of the exponential function in the choice model, a wrong value for μ might lead to full deterministic or full stochastic probabilities. For calibration purposes it is good to ensure a good value for μ is necessary. In this case study two values are used for testing. The value of -0.008 is used for a deterministic simulation. The value of -0.00002 is used to try more stochastic simulations.

Calculation of α

As described in paragraph 7.7.2, the optimal value of α for combining two successive iterations is unknown. Two types are used7.7. The 'MSA' type refers to successive averages in which α is given by

$$\alpha_i = \frac{1}{i} \tag{8.1}$$

The 'Replace'-method is based on a fixed value of 1.0 for α in each iteration.

Number of iterations

The number of iterations used in the scenarios is depending on the value of each scenario. A large number of iterations is used for modelling important scenarios. A lower number is used for some additional analysis.

Definition of initial probabilities

For the first iteration no prior route costs are available. Two options are available. The first option is to use instantaneous free flow travel costs. The second option (uniform spread) is to use a uniform probability spread.

Scenario overview

Table 8.1 gives an outline of the used parameters sets.

Scenario	Choice model	μ	Iteration method	# iterations	First iteration choices
1	PCL	-0.008	Replace	15	Uniform spread
2		-0.008	MSA		
3		-0.00002	Replace	10	
4		-0.00002	MSA		

Table 8.1 Scenarios used for case study

8.3 Results

The results of the scenario runs are displayed in appendix D. For each scenario a graph is presented that shows the duality gap per departure interval over the iterations.

8.3.1 Comparison of scenarios by spread parameter

The first two scenarios differ from the second two in terms of the used spread parameter. As expected, the lower (absolute) value of μ leads to a more stochastic situation in which less iterations are needed and the model converges faster, but keeps a higher duality gap.

8.3.2 Comparison of scenarios by iteration method

The iteration method definitely influences the result of the test scenarios. For the replace method, the converge rate is fast in the first 5 iterations. Then an oscillating effect occurs and no further convergence is reached. The method of successive averages however converges in about 5 iteration and then reaches a stable state without oscillating.

8.3.3 Duality gap of different intervals

All scenarios show a reduction in the duality gap for all departure intervals, even those for which the route choice fractions are fixed. For the deterministic scenarios the duality gap of the objective interval is optimised to have the lowest value of all duality gaps. In the more stochastic scenarios the objective interval can not be optimised to such a level.

8.4 Discussion

It was possible to test only a limited number of scenarios. The results from those tests however indicate that the proposed limited forecasting method leads to expected results: the duality gap converges to a stable state. For the test scenarios only a few iterations were enough to give good indications of the final solution.

The two proposed methods for combining iterations both work. However, the method of successive averages gives significant better results than the replace-method. The latter tends to oscillate, while the former really converges to a stable situation. Further, the MSA approach converges faster and to a lower value of the duality gap.

The influence of the spread parameter μ is like expected. Deterministic models lead to a lower duality gap than stochastic models.

8.5 Value of case study

The case study presented here uses only a small network and the approach used does not include congestion. Although the results from the case study are hopeful, it should be noted that they do not imply that the approach is successful for large scale networks and/or congested networks. Further analysis is needed to see if larger models and time instances further ahead in time (when the load on the network becomes larger) show comparable results.

8.6 Conclusion

Tests have been performed on a small model network with a limited time (and thereby limited network load). Four scenarios have been run to see how the iterative procedure of route choice and network loading leads to a stable situation. Results indicate that all parameters influence the result as expected beforehand. Additional (large scale) research is needed to see how well the used approach performs on large scale (and loaded) networks.

9 Conclusions

9.1 Brief summary

Research objective

In the first chapter the objective of this research was formulated as follows and was supported by four research questions.

> *"The aim of the study is to develop a route choice model as an extension for current macroscopic DNL models, taking into account the interdependence of route choice and network loading."*

Summary

A framework was developed wherein existing DNL models can be used for traffic flow propagation, while the framework has a route choice model and uses an iterative approach with sequential route cost calculation, convergence check, route choice and dynamic network loading.

The route choice problem is presented as discrete choice problem. Five route choice models mentioned in the literature (MNL, CNL, PCL, PSL and C-Logit) have been theoretically described. A large scale network is used to generated routesets for a sample of origins and destinations (26 x 26 zones). From the literature a probit simulation technique is adopted and implemented in Matlab.

Route choice probabilities are estimated for all routes (2148 in total, of which 2010 significant), based on an arbitrary spread parameter. The Logit-based route choice models are then calibrated against a random sample of routesets and validated against all routesets (538 relevant sets in total). The validation process included an analysis of the model performance to characteristic of the routeset. From this analysis the PCL model is chosen as applicable for model use.

Interaction between route choice and dynamic network loading has been investigated. A model is made for combining this model components. This model uses a new dynamic equilibrium definition, since the existing definitions (DUE and Boston equilibrium) were too rigid and not applicable for all model purposes.

The framework is implemented in the OmniTRANS environment and linked to Ruby scripting. Needed import and export to Matlab have been developed. A case study has been performed to test how well the proposed framework performs. Apart from the long model run time the results were promising: all model parameters seem to have the planned effects. Further the new equilibrium method leads to stable conditions.

9.2 Conclusions

Main advances on the research of route choice models

a) In the literature route choice models are approached on a very theoretic base. This research is one of the few in which the models have been applied on a large scale realistic network. It turns out that the models are applicable in such situations and are preferred above on-the-fly path searching from a computational point of view.

b) Five GEV models (MNL, CNL, PCL, PSL and C-Logit) can to some degree be calibrated on the level of a small network. This means that no estimation process on OD-pair level is needed. However, it turns out that the model performance after calibration depends on the characteristics of the routeset for which the model is applied. There are strong relationships between overlap, route costs and model performance.

Main advances for dynamic traffic modelling

c) It is possible to extend existing dynamic network loading models with route generation, filtering and route choice without too many adjustments. A framework is proposed that can be used as a starting point for this adjustment process.

d) The existing dynamic traffic equilibria can be replaced by one single flexible dynamic traffic equilibrium. This model allows to represent traveller behaviour in a more realistic way then the current models, because it allows ‘intermediate’ equilibria (between instantaneous equilibrium and full dynamic equilibrium). As far as known, such an equilibrium is not mentioned in the literature before.

e) The new equilibrium is equipped with a mathematical formulation and solution scheme. The solution algorithm uses a double loop structure for better representation of travel behaviour. A case study with this model indicates that the results are promising if the parameters suit the model application.

9.3 Further research

During the research the following issues have been identified for further research.

f) Departure time modelling is currently not part of the framework. Research can be done to see if the framework can be altered to support departure time choice modelling. Including this functionality would further improve the power of the DTA framework as modelling tool.

g) An empirical research can be conducted to see to what extent route choice in real life follows the limited forecasting approach. This requires data on followed routes and – if possible – decisions made during trip making. Ideally would be to see what route travellers tend to follow while making the trip.

h) Additional tests on larger routesets are needed to see to what extent routeset characteristics determine what route choice model is best to use.

i) Additional simulations are needed to see how well the inner loop method performs on congested networks and with varying forecasting horizons. Especially the overall converge rate and final solution (in terms of duality gap size) are interesting issues subject to parameters like forecasting horizon, number of inner loops and route choice interval.

j) The model functionality of the proportionality factor for modelling multiple types of traveller behaviour and cost perception has to be further investigated. It is believed that the proposed method introduces flexibility and better fit to real world data, but at the same time might lead to an increase of model complexity.

k) One final main issue for further research is the possibility to model adaptive route choice. The introduction of subroutes is likely to require a modification of routeset generation, routeset filtering and the route choice module. It might however lead to significant improvement of the model.

Bibliography

- Bekhor, S. & Toledo, T. (2005). "Investigating path-based solution algorithms to the stochastic user equilibrium problem". In: Transportation Research B, Volume 39, pp. 279-295.

- Ben-Akiva, M. & Lerman, S.R. (1985). "Discrete choice analysis: Theory and Application to Travel Demand". London: MIT Press.

- Berkum, E.C. van (2007). "Macroscopic simulation: METANET". Enschede: Centre for Transport Studies, University of Twente.

- Bierlaire, M. & Frejinger, E. (2005). "Route choice models with subpath components". Conference paper for the 5th Swiss Transport Research Conference. Lausanne: Ecole Polytechnique Fédérale de Lausanne.

- Bierlaire, M. (2005). "Introduction to advanced discrete choice models". Lausanne: Ecole Polytechnique Fédérale Lausanne.

- Bliemer & Bovy (2008). "Impact of Route Choice Set on Route Choice Probabilities". Paper prepared for the 87th Annual Meeting of the Transportation Research Board January 2008, Washington D.C.

- Bliemer (2001). "Analytical dynamic trffic assignment with interacting user-classes". Delft: TRAIL Research School / Delft University Press.

- Bottom, J. & Ben-Akiva, M. & Bierlaire, M. & Chabini, I. & Koutsopoulos, H. & Yang, Q. (1999). "Investigation of route guidance issues by simulation with DynaMIT".

- Bovy, P.H.L. & Stern, E. (1990). "Wayfinding in Transport Networks". Dordrecht: Kluwer Academic Publishers.

- Bovy (1990). "Toedeling van verkeer in congestievrije netwerken". Delft: TRAIL Research School / Delft University Press.

- Clark, P., Culley, R. and Davidson, P. (2008). "Assigning long trips". Paper prepared for the European Transport Conference, October 2007, Noordwijkerhout

- Daganzo, C., Bouthelier, F. and Sheffi, Y. (1977). "Multinomial Probit and Qualitative Choice: a computationally efficient algorithm". In: Transportation Science, Volume 11, Issue 4, pp. 338-358.

- Dijkstra, E.W. (1959). “A note on two problems in connexion with graphs”. In: Numerische Mathematik, Volume 1, pp. 269-271.
- Fiorenzo-Catalano, M.S. (2007). “Choice set generation in multi-modal transportation networks”. Delft: TRAIL Research School / Delft University Press.
- Frejinger, E., Bierlaire, M. (2007). “Capturing correlation with subnetworks in route choice models”. In: Transportation Research B, Volume 41, pp. 363-378.
- Friesz, T.L., Bernstein, D., Smith, T.E., Tobin, R.L. and Wie, B.W. (1993). “A Variational Inequality Formulation of the Dynamic Network User Equilibrium Problem”. In: Operations Research, Volume 43, pp. 1120-1136.
- Han, S. (2000). “Dynamic Traffic Modelling and Dynamic Stochastic User Equilibrium Assignment for General Road Networks”. London: Centre for Transport Studies,UCL.
- Hoogendoorn-Lanser, S. & Nes, R. van & Bovy, P. (2004). “Path-size modeling in multi-modal route choice analysis”. Paper prepared for the 84th Annual Meeting of the Transportation Research Board January 2005, Washington D.C.
- Kuwahara, M. and Akamatsu, T. (2001). “Dynamic User Optimal Assignment with physical queues for a many-to-many OD pattern”. In: Transportation Research B, Volume 35, pp. 461-479.
- Ministry of Transport and Public Works (2004). “Nota Mobiliteit: Towards reliable and predictable accessibility”. Den Haag: Ministry of Transport and Public Works.
- Ortúzar, J. de Dios & Willumsen, L.G. (2001). “Modelling Transport, third Edition”. London: John Wiley & Sons Ltd.
- Patriksson, M. (1994). “The traffic assignment problem: Models and methods”. Linköping: Linköping Institute for Technology.
- Peeta, S. & Ziliaskopoulos, A.K. (2001). “Foundations of Dynamic Traffic Assignment: The past, the present and the future”. In: Networks and Spatial Economics, Volume 1, pp. 233-265. Dordrecht: Kluwer Academic Press.
- Ramming, M.S. (2002). “Network knowledge and route choice”. Cambridge: Massachusetts Institute of Technology.

- Ran, B., Boyce, D.E. and LeBlanc, L.J. (1993). “A new class of instantaneous dynamic user-optimal traffic assignment models”. In: Operations Research, Volume 41, Issue 1, pp. 192-202.

- Rosa, A. and Maher, M. (1999). “Stochastic user equilibrium traffic assignment with multiple user classes and elastic demand”. Edinburgh: Napier University.

- Sheffi, Y. (1985). “Urban Transportation Networks: Equilibrium Analysis with mathematical programming methods”. Englewood Cliffs: Prentice-Hall Inc.

- SWOV (2003). “Routekeuze in een wegennet”. Leidschendam: SWOV.

- Train, K. (2002). “Discrete choice models with simulation”. New York: Cambridge University Press.

- Yperman, I. (2007). “The Link Transmission Model for Dynamic Traffic Assignment”. Leuven-Heverlee: Katholieke Universiteit Leuven.

- Wardrop, J.G. (1952). “Some theoretical aspects of road traffic research”. In: Proceedings of the Institution of Civil Engineers, Part II 1, pp. 325-378.

Indirect references

- Ben-Akiva, M. & Bergman, M.J. & Daly, A.J. & Ramaswamy, R. (1984). “Modeling inter-urban route choice behaviour”. In: Proceedings of the 9th international symposium on transportation and traffic theory. *As cited in Bovy & Stern (1990).*

- Benshoof, J.A. (1970). “Characteristics of drivers’ route selection behaviour”. In: Traffic Engineering and Control, Volume 11, Issue 12, pp. 604-606,609. *As cited in Bovy & Stern (1990).*

- Dial, R.B. (1971). “A probabilistic multipath traffic assignment model which obviates the need for path enumeration”. In: Transportation Research, Volume 5, pp. 83-111. *As cited in Fiorenzo-Catalano (2007).*

- Hoogendoorn-Lanser (2005). “Modelling travel behaviour in multi-modal networks”. *As cited in Fiorenzo-Catalano (2007).*

- Jansen, G.R.M. and Den Adel, D.N. (1987). “Routekeuze van automobilisten: een onderzoek naar kwalitatieve keuzefactoren”. Delft: Delft University Press. *As cited in Bovy & Stern (1990).*

- Stern, E. and Leiser, D. (1988). “Levels of spatial knowledge and urban travel modelling”. In: Geographical Analysis. Volume 20, Issue 2, pp. 140-155. *As cited in Bovy & Stern (1990).*

- Szeto, W.Y. (2003). “Dynamic traffic assignment: formulations, properties, and extensions”; Ph.D. Thesis. Hong Kong: Hong Kong University of Science and Technology. *As cited in Yperman (2007).*

- Vovsha, P. & Bekhor, S. (1998). “The Link-Nested Logit Model of Route Choice: Overcoming the Route Overlapping Problem.” Transportation Research Record, 1645, 133. *As cited in Ramming (2002).*

Appendix A Conceptual framework diagram

This page can be folded out for viewing while reading. The figure depicts the framework introduced in chapter 3 and of which elements are covered in the chapter 4, 5, 6, 7 and 8.

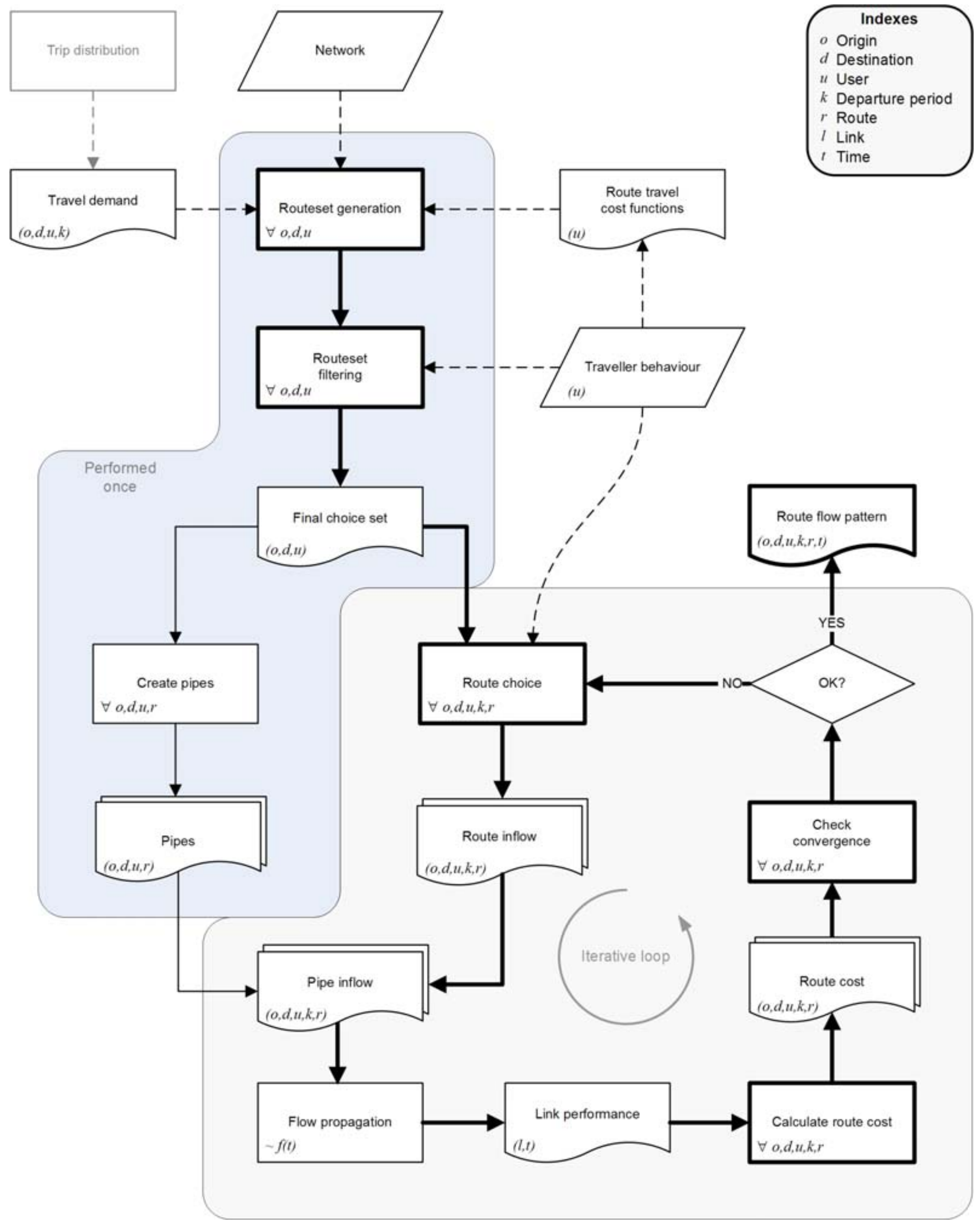


Figure A.1 Conceptual framework

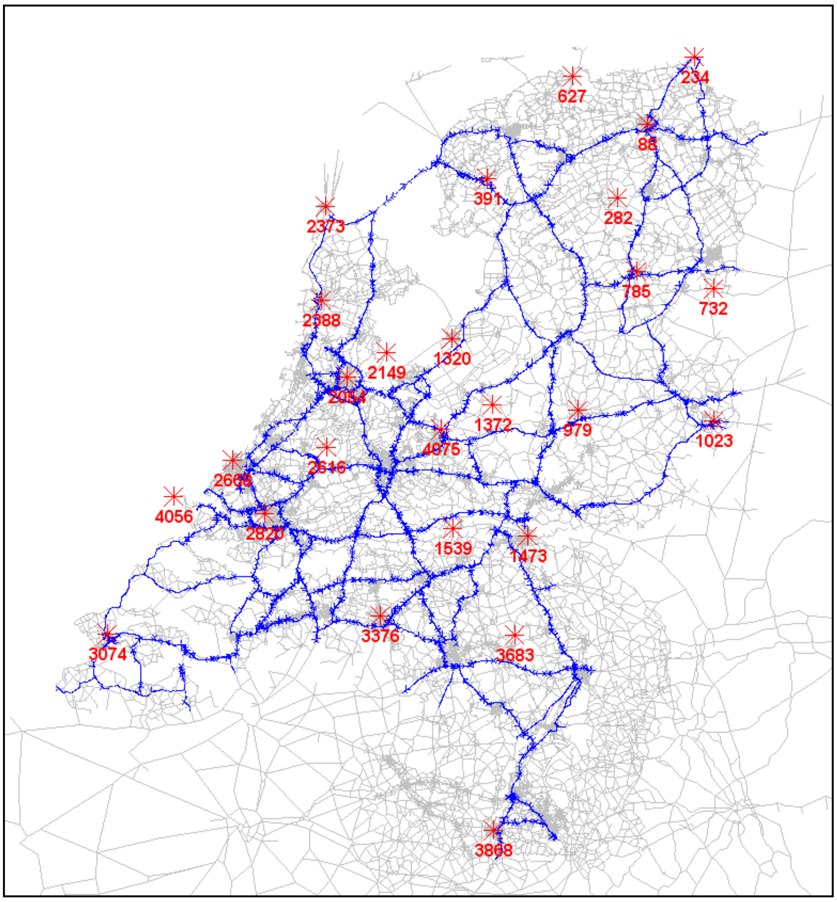
The indices indicate for which dimensions the action has to be performed or data is specified. The thick outlined blocks are covered in this research. Thick arrows indicate the main model data flow.

Appendix B Input parameters routeset generation and filtering

Zones used for routeset generation

Number	Description	Number	Description	Number	Description
88	Groningen	1320	Lelystad	2668	Den Haag
234	Delfzijl	1372	Garderen	2820	Rotterdam
282	Dwingeloo	1473	Nijmegen	3074	Middelburg
391	Sneek	1539	Alphen (Gld)	3376	Tilburg
627	Lauwersoog	2054	Amsterdam	3683	Asten
732	Bourtange	2149	Marken	3868	Maastricht
785	Hoogeveen	2373	Den Helder	4056	Tweede Maasvlakte
979	Deventer	2388	Alkmaar	4075	Amersfoort Vathorst
1023	Enschede	2616	Boskoop		

Map of network and zones



Generation parameters

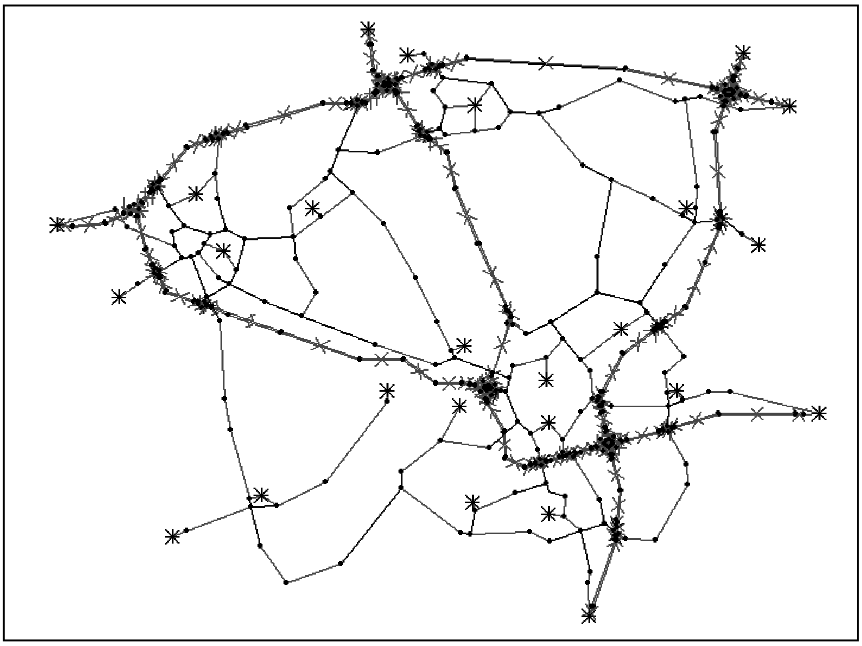
Initial variance for randomisation	0.09
Increase of variance	0.02
Maximum variance	0.30
Number of unsuccessful iterations before increasing variance	3
Maximum number of iterations	50

Filtering parameters

α	Maximum overall detour	1.90
Δ	Maximum overlap	0.60
ω_{max}	Maximum section detour	2.00
ω_{min}	Minimal section detour	0.01
S	Maximum size of choiceset	6

Appendix C Network used for case study (chapter 8)

Network overview "Zuid-Meerendal"

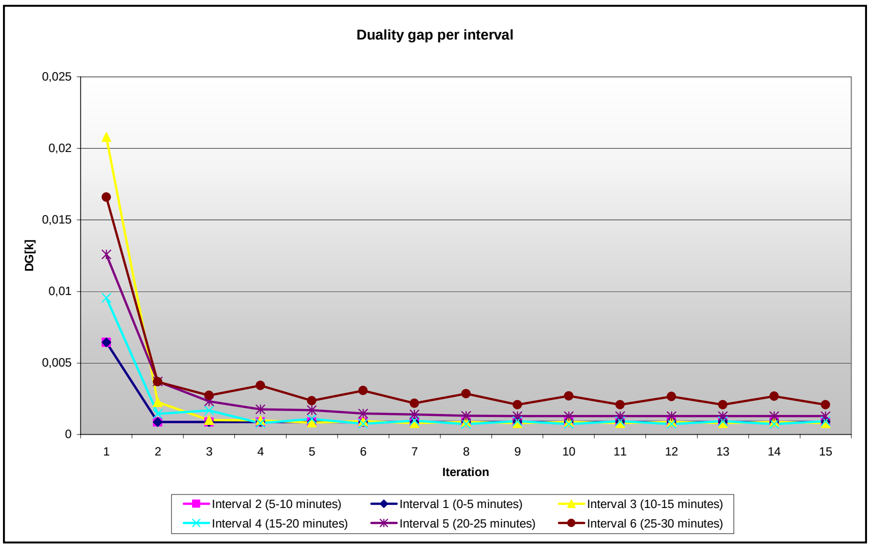


Appendix D Results case study

Scenario 1

Spread parameter $\mu = -0.008$

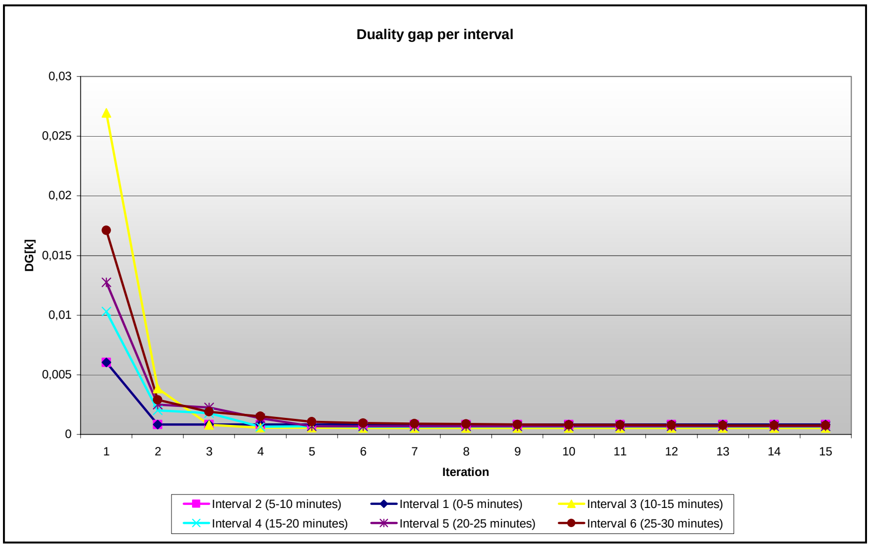
Route choice update strategy: replace



Scenario 2

Spread parameter $\mu = -0.008$

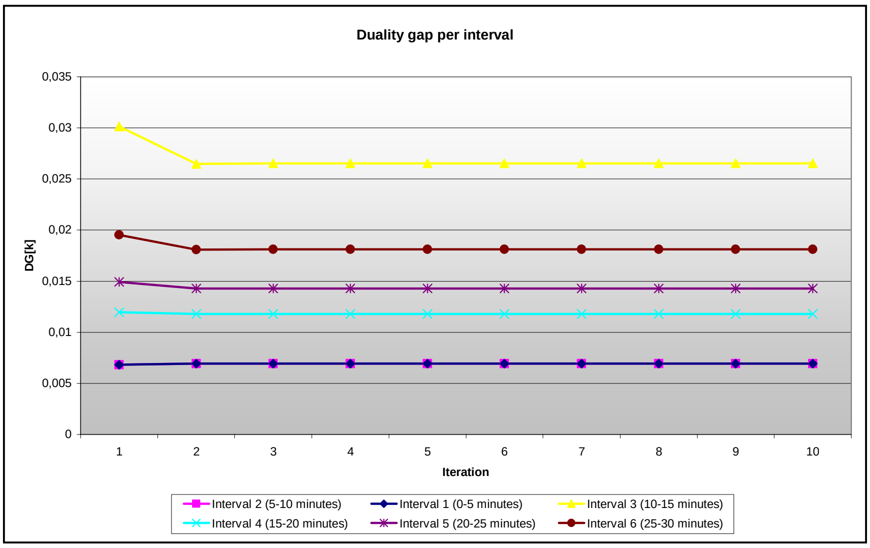
Route choice update strategy: MSA



Scenario 3

Spread parameter $\mu = -0.00002$

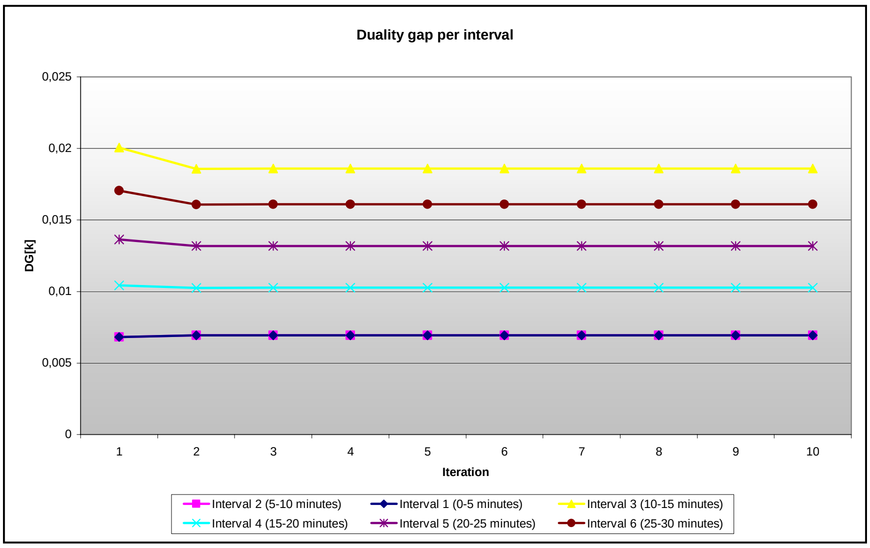
Route choice update strategy: replace



Scenario 4

Spread parameter $\mu = -0.00002$

Route choice update strategy: MSA



Appendix E List of figures

Appendix F List of tables